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HASSE-WITT MATRICES FOR THE FERMAT CURVES OF PRIME DEGREE

JOSEP GONZÁLEZ

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Abstract. We study a class of curves which includes the hyperelliptic curves and the Fermat curves of prime degree. We compute their Hasse-Witt matrix when the curves are defined over an algebraically closed field of positive characteristic. In particular, we get a formula for the Hasse-Witt invariant of the Fermat curves at each prime not equal to the degree. These invariants depend only on residue degrees.

We show that there are infinitely many Fermat curves for which there exists a set of primes p with positive density such that the geometric fibre at p of the Fermat Jacobian is not isogenous to a power of a supersingular elliptic curve, but the Hasse-Witt invariant at p is equal to zero.

1. Introduction. Let C be a non-singular projective curve of genus $g>0$, defined over an algebraically closed field k of characteristic $p>0$. Hasse and Witt [Ha34], [Ha-Wi36] determined the maximum number, $r(C)$, of cyclic unramified independent extensions of degree p of the function field $k(C)$. This number is called the Hasse-Witt invariant of the curve.

Serre [Se58] characterized the Hasse-Witt invariant by means of the action of the absolute Frobenius F^* on the first cohomology group of the curve:

$$r(C)=\dim_{\boldsymbol{F}_p} H^1(C, \mathcal{O})^{F^*}=\dim_k \mathrm{Im}(F^*)^g .$$

The curve C is said to be *ordinary* if $r(C)=g$. When $g=1$, it is said to be *supersingular* if $r(C)=0$.

The computation of this invariant for elliptic curves is well known. Manin [Ma62] computed the Hasse-Witt invariant for hyperelliptic curves by means of the matrix of the Cartier operator acting on an explicit basis of regular differentials. The structure of the p-divisible groups arising from Fermat curves over finite fields of characteristic p was studied by Yui in [Yu80]; arithmetical invariants for Fermat curves and Fermat varieties were also consider in [Yu86] and Toki [To88]. In this paper we compute the Hasse-Witt invariant for curves of a special family, which includes, as particular cases, the hyperelliptic curves and the Fermat curves. We use for it the absolute Frobenius operator acting on an explicit basis of repartitions. The advantage of using the absolute Frobenius instead of the Cartier operator will be apparent when we compute the matrix

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