Tôhoku Math. J.
49 (1997), 449–484

IRREDUCIBLE CONSTANT MEAN CURVATURE 1 SURFACES IN HYPERBOLIC SPACE WITH POSITIVE GENUS

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(Received June 9, 1995, revised May 15, 1997)

Abstract. In this work we give a method for constructing a one-parameter family of complete CMC-1 (i.e. constant mean curvature 1) surfaces in hyperbolic 3-space that correspond to a given complete minimal surface with finite total curvature in Euclidean 3-space. We show that this one-parameter family of surfaces with the same symmetry properties exists for all given minimal surfaces satisfying certain conditions. The surfaces we construct in this paper are irreducible, and in the process of showing this, we also prove some results about the reducibility of surfaces.

Furthermore, in the case that the surfaces are of genus 0, we are able to make some estimates on the range of the parameter for the one-parameter family.

1. Introduction. Recently, new examples of immersed CMC-1 surfaces of finite total curvature in the hyperbolic 3-space $H^3(-1)$ of curvature -1 have been found (cf. [UY1], [UY2], [S], and [UY4]). One method used to make these new examples is the following: The set of all conformal branched CMC-c (i.e. constant mean curvature c) immersions in $H^3(-c^2)$ of finite total curvature with hyperbolic Gauss map G defined on a compact Riemann surface $\bar{M}$ corresponds bijectively to the set of conformal pseudometrics of constant curvature 1 with conical singularities on $\bar{M}$. (cf. [UY4].) By the work of Small [S], this correspondence can be explicitly written when the immersion can be lifted to a null curve in $PSL(2, \boldsymbol{C})=SL(2, \boldsymbol{C})/\{\pm 1\}$. This gives a method for constructing new examples. However, to construct non-branched CMC-c surfaces is still difficult, because the method above does not give any control over branch points.

In this paper we use a new method to construct new examples without branch points, which have higher genus, many symmetries, and embedded ends. More precisely, we prove that for each complete symmetric finite-total-curvature minimal surface in $\boldsymbol{R}^3$ with a non-degenerate period problem, there exists a corresponding one-parameter family of CMC-1 surfaces in $H^3(-1)$. We define the terms "symmetric" and "non-degenerate" later. To prove the existence of these corresponding one-parameter families, we begin by using a small deformation from the original minimal surface in $\boldsymbol{R}^3$, preserving its (hyperbolic) Gauss map G and Hopf differential Q. This gives us CMC-c surfaces in $H^3(-c^2)$, for $c\approx 0$. Finally, we rescale the surfaces into CMC-1

1991 *Mathematics Subject Classification*. Primary 53A10; Secondary 53A35, 53C42.

This research was supported in part by Grant-in-Aid for Scientific Research, the Ministry of Education, Science, Sports and Culture, Japan, and by a fellowship from the Japan Society for the Promotion of Science.