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THE FUNCTOR OF A TORIC VARIETY WITH ENOUGH INVARIANT EFFECTIVE CARTIER DIVISORS

TAKESHI KAJIWARA

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Abstract. The homogeneous coordinate ring of a toric variety was first introduced by Cox. In this paper, we study that of a toric variety with enough invariant effective Cartier divisors in detail. Here a toric variety is said to have enough invariant effective Cartier divisors if, for each nonempty affine open subset stable under the action of the torus, there exists an effective Cartier divisor whose support equals its complement. Both quasi-projective toric varieties and simplicial toric varieties have enough invariant effective Cartier divisors. In terms of the homogeneous coordinate ring, we describe the data needed to specify a morphism from a scheme to such a toric variety. As a consequence, we generalize a result of Cox, one of Oda and Sankaran, and one of Guest concerning data on morphisms.

Introduction. Let k be a field, N a free $\boldsymbol{Z}$-module of rank r, M the $\boldsymbol{Z}$-module dual to N, $T:=\boldsymbol{G}_m\otimes N$ the algebraic torus of dimension r corresponding to N, and $\varDelta$ a (finite) fan of $N_{\boldsymbol{Q}}$. Let $X_\varDelta$ be the toric variety associated to $\varDelta$, D_ρ the closure of the T-orbit corresponding to a one-dimensional cone $\rho\in\varDelta$, $\sigma(1)$ the set of one-dimensional cones contained in a cone $\sigma\in\varDelta$, and $\mathrm{Pic}(\varDelta)_{\geq 0}$ the monoid of linear equivalence classes of invariant effective Cartier divisors. A toric variety $X_\varDelta$ is said to have enough invariant effective Cartier divisors if, for each cone $\sigma\in\varDelta$, there exists an effective T-invariant Cartier divisor D with $\mathrm{Supp}\, D=\bigcup_{\rho\notin\sigma(1)}D_\rho$. Both quasi-projective toric varieties and simplicial toric varieties have enough invariant effective Cartier divisors (cf. Remark 1.6(3)).

Cox [1] introduced two homogeneous coordinate rings of a toric variety $X_\varDelta$: one is the monoid algebra S of the monoid of effective T-invariant Weil divisors with Chow-grading, while the other is the subring $S_\varDelta$ of S with Pic-grading (see [1, p. 19, p. 35]). He constructed in [1] the toric variety $X_\varDelta$ as the quotient of an open subscheme of $\mathrm{Spec}\, S$, and described in [2, Theorem 1.1] the data needed to specify a map from a scheme to an arbitrary *smooth* toric variety in terms of its homogeneous coordinate ring.

The purpose of this paper is to generalize Cox's description to one for an arbitrary toric variety with enough invariant effective Cartier divisors by studying the latter homogeneous coordinate ring in detail (cf. Theorem 3.4 and Theorem 4.3).

The contents of this paper are as follows:

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