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EISENSTEIN SERIES ON WEAKLY SPHERICAL HOMOGENEOUS SPACES OF $GL(n)$

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Abstract. A homogeneous space of a reductive group is called weakly spherical if the action of some proper parabolic subgroup is prehomogeneous. We associate Dirichlet series with weakly spherical homogeneous spaces defined over the rational number field and prove their functional equations in the case where the space under consideration is a homogeneous space of the general linear group.

Introduction.

0.1. Let G be a connected reductive algebraic group and P a proper parabolic subgroup. A homogeneous space $X=G/H$ of G is said to be *P-spherical* if there exists a Zariski-open P-orbit in X. In this case we also say that (G, H, P) is a *spherical triple*. We call X *spherical* (resp. *weakly spherical*) if X is B-spherical (resp. P-spherical) for a Borel subgroup B (resp. for some proper parabolic subgroup P). It is well-known that symmetric spaces are spherical (cf. [V]).

In [S3], [S5], [S6] and [HS], we introduced generalized Eisenstein series attached to (not necessarily Riemannian) symmetric spaces with $\boldsymbol{Q}$-structure and proved that, in a number of cases, the generalized Eisenstein series have nice analytic properties (analytic continuation, functional equations) similar to the properties of the Selberg-Langlands Eisenstein series. However, in the definition of the generalized Eisenstein series given in [HS], the assumption that a homogeneous space in question is a symmetric space is irrelevant and what is essential is that it is (weakly) spherical. Therefore one can naturally ask to what extent the results in the papers cited above can be generalized to general weakly spherical homogeneous spaces.

In [S7], we have shown that the theory of zeta functions in one variable associated with prehomogeneous vector spaces developed in [SS] gives an affirmative answer to the question above in the case where $G=GL(n)$ and P is its maximal parabolic subgroup.

In the present paper, we consider the case where G is a product of several general linear groups and P is its (not necessarily maximal) parabolic subgroup.

0.2. Set $G=GL(m_1)\times\cdots\times GL(m_l)$ and $\Gamma=SL(m_1, \boldsymbol{Z})\times\cdots\times SL(m_l, \boldsymbol{Z})$. Let P be a standard parabolic subgroup and H a reductive $\boldsymbol{Q}$-subgroup of G such that $X=G/H$ is P-spherical. Let Ω be the open P-orbit in X. We put $\mathfrak{a}^*_{P,\boldsymbol{C}}=\mathrm{Hom}_{\boldsymbol{Q}}(P, \boldsymbol{G}_m)\otimes\boldsymbol{C}$. Then

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