

REDUCTION OF THE HERMITIAN-EINSTEIN EQUATION ON KÄHLERIAN FIBER BUNDLES

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Abstract. The technique of dimensional reduction of an integrable system usually requires symmetry arising from a group action. In this paper we study a situation in which a dimensional reduction can be achieved despite the absence of any such global symmetry. We consider certain holomorphic vector bundles over a Kahler manifold which is itself the total space of a fiber bundle over a Kahler manifold. We establish an equivalence between invariant solutions to the Hermitian-Einstein equations on such bundles, and general solutions to a coupled system of equations defined on holomorphic bundles over the base Kahler manifold. The latter equations are the Coupled Vortex Equations. Our results thus generalize the dimensional reduction results of García-Prada, which apply when the fiber bundle is a product and the fiber is the complex projective line.

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1. Introduction. Techniques involving dimensional reduction are important in many areas of mathematical physics when one is looking at solutions to partial

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