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A NOTE ON THE FACTORIZATION THEOREM OF TORIC BIRATIONAL MAPS AFTER MORELLI AND ITS TOROIDAL EXTENSION

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Abstract. Building upon a work of Morelli, we give a coherent presentation of Morelli's algorithm for the weak and strong factorization of toric birational maps. We also discuss its toroidal extension, which plays a crucial role in the recent solutions by Włodarczyk and Abramovich-Karu-Matsuki-Włodarczyk of the weak factorization conjecture of general birational maps.

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0. Introduction. This paper is a result of series of seminars held by the authors during the summer of 1997 and continued from then on, toward a thorough understanding of the following weak and strong factorization theorem of toric birational maps by Morelli [Morelli1] (cf. [Włodarczyk1]).

THEOREM 0.1 (Factorization Theorem for Toric Birational Maps). *Every proper and equivariant birational map $f : X_\Delta \dashrightarrow X_{\Delta'}$ ("proper" in the sense of ⌈Iitaka⌉) between two nonsingular toric varieties can be factored into a sequence of blowups and blowdowns with smooth centers which are the closures of orbits.*

If we allow the sequence to consist of blowups and blowdowns in any order, then the factorization is called *weak.*

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