Tohoku Math. J.
51 (1999), 433–446

A CONSTRUCTION OF K-CONTACT MANIFOLDS BY A FIBER JOIN

TSUTOMU YAMAZAKI

(Received March 21, 1997, revised June 21, 1999)

Abstract. In this paper we introduce a process of making a fiber join of regular K-contact manifolds and then construct some explicit examples of K-contact flows which generate contact transformations of a torus. We also discuss the equivalence of these examples.

1. Introduction. A contact flow φ_t is a flow which is generated by the Reeb vector field of a contact manifold (M, α). It preserves the contact form α and the contact plane field $\ker\alpha$. A contact flow φ_t is called a *K-contact flow* if there exists a metric g on M such that φ_t is an isometry. In this case the triple (M, α, g) is called a *K-contact manifold* ([2, 3]).

Suppose we are given a K-contact manifold (M, α, g). If M is compact, the closure of a K-contact flow $\{\varphi_t \mid t \in \boldsymbol{R}\}$ in the isometry group of (M, g) makes a compact connected abelian Lie group, hence isomorphic to T^k for some integer k. Clearly this action of the torus T^k also preserves α and g. Thus a compact K-contact manifold (M, α, g) has a T^k-action which preserves both α and g. We will see that this property of T^k-action on a contact manifold characterizes the "K-contactness" and k satisfies $1 \leq k \leq n+1$ when $\dim M = 2n+1$ (see Proposition 2.1). We call (M, α, g) with this T^k-action a K-contact manifold *of rank k*. A typical class of examples of K-contact manifolds of rank 1 is a family of *regular* K-contact manifolds (M, α, g). A regular contact manifold (M, α) consists of a pair of a principal S^1-bundle M over a symplectic manifold (W, ω) and a connection one-form α. A metric g is given by $g = \pi^* g_W \oplus (\alpha \otimes \alpha)$, where g_W is a Riemannian metric compatible with ω and π is the bundle projection $M \to W$ (see Example 2.4).

In this paper we will present a method of constructing a K-contact manifold of rank $k \geq 2$ out of K-contact manifolds of rank 1 by making use of join construction in topology.

Let $(M_0, \alpha_0, g_0), \dots, (M_n, \alpha_n, g_n)$ be regular K-contact manifolds and L_j an associated complex line bundle of $M_j \to W$ for each j $(j = 0, 1, \dots, n)$. From these we construct a K-contact manifold $(M_0 *_f \cdots *_f M_n, \beta_\lambda, g_\lambda)$ of rank $n+1$. Here $M_0 *_f \cdots *_f M_n$ is the unit sphere bundle $S(L_0 \oplus \cdots \oplus L_n)$ and β_λ is a contact form with a parameter $\lambda = (\lambda_0, \dots, \lambda_n) \in \boldsymbol{R}^{n+1}$. We call the resulted K-contact manifold a *fiber join* of $(M_0, \alpha_0, g_0), \dots, (M_n, \alpha_n, g_n)$.

Applying a fiber join construction to three dimensional regular K-contact manifolds, we obtain infinitely many distinct K-contact structures on $\Sigma_g \times S^{2n+1}$ and $\Sigma_g \tilde{\times} S^{2n+1}$ (Σ_g is a closed Riemannian surface of genus g) which are not T^{n+1}-equivariant. Namely, we obtain the following:

1991 *Mathematics Subject Classification*. Primary 53C12; Secondary 5315, 58F25.