NOTES ON BANACH SPACE (X): VITALI-HAHN-SAKS' THEOREM AND K-SPACES.[*]

By

Masahiro Nakamura.

Since H. Freudenthal [4] established the spectral respresentation for the vector-lattices, the closed analogy between the theory of the vector-lattices and that of the additive set functions are pointed out by several authors. As its consequence the well-known theorem of Radon-Nikodym are abstractly handled.

This note[1] lies in this direction, and firstly we prove the Hahn-decomposition theorem and the Vitali-Hahn-Saks theorem of the additive set functions for the Banach-lattices. The former, as proved by G. Birkhoff [2], is already known in strictly monotone Banach lattices. But, as will be seen in the following, if we restrict the linear functional suitably, then Birkhoff's proof is applicable in some more general cases. The later is obtained by T. Ogasawara [12] and H. Nakano [10] independently. Ogasawara's proof depends on his representation theory and concrete case of the Vitali-Hahn-Saks theorem. On the other hand, Nakano's proof is fine but does not contain the classical theory of the additive set functions. In this note, we prove it containing both cases, using the method of S. Saks [13]. Hence it may be observed with some interest.

As an application of the above theorems, we prove in §3 some structure theorems due to T. Ogasawara [11] on K-spaces. In §2, we will prove some lemmas, which are due to T. Ogasawara [11], [12]. We gave a sketch of proof, which does not depends on the concrete representation theory, and then seems to be somewhat simpler than those of T. Ogasawara.

Throughout this note, we use the terminologies of the text books of G. Birkoff [2] and S. Banach [1] without any explanation. But there is one different point which is the notion of the "ideal" of the vector lattices. We use it here as "closed admissible l-ideal" in [3] or "complemented normal

---