NOTES ON FOURIER ANALYSIS (XX): ON THE RIESZ LOGARITHMIC SUMMABILITY OF THE DERIVED FOURIER SERIES.*)

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1. Let $f(x)$ be an integrable function with the period 2π and its Fourier series be

(1) $$f(x) \sim \frac{1}{2} a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx).$$

If we differentiate the series term by term, we get

(2) $$\sum_{1}^{\infty} n(-a_n \sin nx + b_n \cos nx),$$

which is said the derived Fourier series of $f(x)$ and denote it by $S'[f]$.

The object of the present paper is to treat the Riesz logarithmic summability of (2).

Concerning the Fourier series Wang has proved the following theorems:

Theorem A. If

$$\lim_{t \to 0} \varphi(t) = s \ (R, \log n, \alpha) \quad (\alpha > 0),$$

then (1) is $(R, \log n, \alpha+\delta)$-summable to s at $t=x$, where δ is any positive number.

Theorem B. If (1) is $(R, \log n, \alpha)$-summable to sum s at $t=x$, then

$$\lim_{t \to 0} \varphi(t) = s(R, \log n, \alpha+1+\delta) \quad (\alpha > 0).$$

We prove analogus theorems concenning derived Fourier series (2), which reads as follows:

Theorem 1. If

$$\psi(t)/t = s \ (R, \log n, \alpha) \quad (\alpha > 0),$$

then (2) is $(R, \log n, \alpha+1+\delta)$-summable to sum s at $t=x$, where δ is any positive number.

Theorem 2. If (2) is $(R, \log n, \alpha)$-summable to sum s at $t=x$ $(\alpha > 1)$, then

$$\lim_{t \to 0} \psi(t)/t = s \ (R, \log n, \alpha+1+\delta)$$

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