NOTES ON FOURIER ANALYSIS (XVIII): ABSOLUTE SUMMABILITY OF SERIES WITH CONSTANT TERMS.*)

By

Gen-ichirô Sunouchi.

The object of this paper is to prove some theorems concering absolute summability systematically. In § 1, key theorems are proved, from which theorems of the remaining sections are derived. One of the key theorems reads as follows: when (x_n) is a given sequence and (y_n) is defined by

$$y_n=a_{n,0}\ x_0+a_{n,1}\ x_1+\cdots\cdots+a_{n,m}\ x_m+\cdots\cdots,$$

where $(a_{n,k})$ is an infinite matrix, then

$$\sum_{n=0}^{\infty}|a_{n+1,m}-a_{n,m}|<M\ (m=1,2,\cdots\cdots)$$

is the necessary and sufficient condition that any $\sum_{n=0}^{\infty}|x_n|<\infty$ implies $\sum_{n=0}^{\infty}|\Delta y_n|<\infty$. By this and the similar key theorems we prove theorems of Mercerian type (in § 3), inclusion relation between absolute Riesz summations of different types (in § 4) and Tauberian theorems (in § 5).

§ 1. Key theorems. Let (x_n) be a sequence of real number and its linear transformation be

$$(1)\qquad y_n=\sum_{k=0}^{\infty}a_{n,k}\ x_k.$$

Theorem 1. In order that any $\sum_{n=0}^{\infty}|x_n|<\infty$ implies $\sum_{n=0}^{\infty}|\Delta y_n|<\infty$, it is necessary and sufficient that

$$(2)\qquad \sum_{n=0}^{\infty}|a_{n+1,m}-a_{n,m}|<M.$$

Proof. Necessity. We have

$$\Delta y_n=y_{n+1}-y_n=\sum_{m=0}^{\infty}(a_{n+1,m}-a_{n,m})\ x_m$$

which is a linear functional on (l). If we put $x\equiv(x_n)\,\varepsilon\,(l)$, $\Delta y_n\equiv U_n(x)$, then $W(x)\equiv\sum_{n=0}^{\infty}|U_n(x)|$ satisfies the assumption of the Bosanquet-Kestelman theorem [2]. Hence we have

$$\sum_{n=0}^{\infty}|U_n(x)|\leqq M\|x\|.$$

If we put $x_n=1\,(n=m)$, $x_n=0\ (n\neq m)$, then we get (2). Thus the necessity

*) Received Oct. 8th, 1947.