NOTES ON FOURIER ANALYSIS (XV)
ON THE ABSOLUTE CONVERGENCE OF TRIGONOMETRICAL SERIES.*)1)

By

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I. Let us conder the trigonometrical series

(1) $$\sum_{n=1}^{\infty} \rho_n \cos (nx-\alpha_n)$$

where $\rho_n \geqq 0$ $(n=1,2,\ldots\ldots)$ and

(2) $$\sum_{n=1}^{\infty} \rho_n=\infty.$$

We have proved that if

(3) $$\rho_n=O(1/n),$$

then the set of points where the series (1) converges absolutely is of α-caqacity zero $(0<\alpha<1)$[2].

We can now prove more precise result;

Theorem[3] If $\rho_n=O(1/n)$ and $\sum_{n=1}^{\infty} \rho_n=\infty$, then we have

(4) $$\lim_{n\to\infty} \frac{\sum_{k=1}^{n} \rho_k |\cos (kx-\alpha_k)|}{\sum_{k=1}^{n} \rho_k}=2/\pi$$

except a set of α-capacity zero $(0<\alpha<1)$.

II. We shall firstly prove the following lemma.

Lemma. If (γ_n) is a sequence of complex quantities such that

$$\sum_{n=1}^{\infty} |\gamma_n|=\infty, \text{ and } |\gamma_n|=O(1/n),$$

then we have

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1) Read before the annual meeting of the Mathematical Society at May, 1946.

2) T. Tsuchikura and S, Yano, Notes on Fourier Analysis (V): Absolute convergence of trigonometrical series, under the press.

3) cf. R. Salem, The absolute convergence of trigonometrical series, Duke Math. Journ., 8 (1941).