ON NORMAL COORDINATES OF A RIEMANN SPACE, WHOSE HOLONOMY GROU FIXES A POINT.*)

By

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§ 1. Consider a Riemann space V_n whose distance ds between two infinitely nearby points is given by

$$ds^2 = g_{\lambda\mu} dx^\lambda dx^\mu, \quad (\lambda,\mu,\nu,\cdots\cdots=1,2\ \cdots\cdots n),$$

where the right hand member is a positive definite quadratic form.

Any normal coordinate system $(\bar{x}^\lambda)$ of V_n with a point O as origin is characterized by the condition that the following equations

$$\overline{\left\{ {\lambda \atop \mu\nu} \right\}} \bar{x}^\mu \bar{x}^\nu = 0 \tag{1}$$

are satisfied at every point in a neighbourhood of O $(\bar{x}^\lambda=0)$ of V_n. Let $\bar{g}_{\lambda\mu}$ be the metric tensor of this coodinate system. According as $\bar{g}_{\lambda\mu}$'s have definite values or not at the origin, we call•the normal coordinate system in consideration *ordinary* or *singular* respectively.

Consider an arbitrary coordinate syatem(x^λ). Then if a point is designated as the origin, one and only one normal coordinate system is determined so that both metric tensors have same values at this point and the transformations of normal coordinate systems with the same origin constitute a linear representation of the original coordinate transformations.

Now, if there is any point such that $|g_{\lambda\mu}|=0$ or some of $g_{\lambda\mu}$'s have indefinite values with respect to some coordinate systems, we say that they are singular points of V_n. If P is not a singular point, there exists at least one coodinate system such that $g_{\lambda\mu}$'s with $|g_{\lambda\mu}| \neq 0$ have definite values at P. Hence the normal coordinate system coresponding to such coordinate system and having P as origin is ordinary. Accordingly, every singular point is characterized by the condition that any normal coordinate system with this point as its origin is necessarily a singular one.

§ 2. Now consider a V_n, whose holonomy group fixes a point O. Consider a normal coordinate system $(\bar{x}^\lambda)$ with O as its origin, then every geodesic issuing from O is expressible by the equations $\bar{x}^\lambda = \xi^\lambda s$ where S is the arc

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