A NOTE ON GENERAL TOPOLOGICAL SPACES.*)

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1. If for any subset A of the fundamental set S we can assign a "closure" $\bar{A}$ satisfying some proper conditions, then the set S is said to be a space. In general there are two methods defining the closure, that is;

(I) When there corresponds a family "neighbourhoods" V_x to every point x in S, $x \in \bar{A}$ is, by definition, that no $V_x \cap A$ is vacuous.

(II) When there is a family of "sequences" $\{x_\alpha\}$ in S(1) for which it is always decided that $\{x_\alpha\}$ converge to x or not; $x \in \bar{A}$ is by definition, that there is a sequence in A convergent to x.

S is said to be a neighbourhood space or convergent space according as it is topologized by a system of neighbourhoods or a family of convergent sequences. When convergence of sequences are suitably defined by means of system of neibourhoods, the neighbourhood space becomes a convergence space. For example, if in a neighbourhood space S convergency of the sequence $\{x_\alpha\}$ is defined by

(III) $\{x_\alpha\}$ converges to x if and only if for each neighbourhood V_x of x, there exists an $\alpha_0 = \alpha_0(V_x)$ such that $\alpha > \alpha_0$ implies $x_\alpha \in V_x$, then S becomes a convergence space.

In this paper we intoduce the notion of "φ-closure" (in Definition 2), by which neighbourhood space turns to the space with "φ-topology". Main results concerning φ-topology are cotained in Theorem 4.

But if we consider some set A such as $\{x_\alpha\} \subset A \subset S$, we obtain many interesting results, for instance, all convergence topologies defined in S is a Boolean algebra(2) by some order relation.

2. Let φ be a set-function on 2^S (=family of all subsets in S) such that

(2, 1) for any subset A in S, $A \subset \varphi(A)$,

(2, 2) $A \subset B$ implies $\varphi(A) \subset \varphi(B)$.

And let ϕ be the class of all such φ.

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(1) For any finite or infinite directed set.

(2) G. Birkhoff, Fund Math., XXVI(1936).