LINEAR TOPOLOGICAL SPACES AND ITS PSEUDO-NORMS.*)

By

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Linear topological spaces were studied by A. Kolmogoroff,[1] J. v. Neumann,[2] H. Hyers[3] and many other authers. Concerning relations among these investigations, J. V. Wehausen[4] proved the equivalency of linear topological spaces of Neumann and Kolmogonoff, and Hyers gave a new defintion of linear topological spaces equivalent to them. After him to any linear topological space we can associate a cernain directed system. When we examine this directed system, we see that the directed system can be replaced by a semi-join-lattice, and the linear topological space is characterized by the family of new topologies which form a semi-join-lattice (§ 2). In § 3 we show that this semi-lattice can be replaced by the semi-meet-attice. The norm of the convex linear topological space satisfies the triangular inequality. But the "Norm" of § 3 does not necessarily satisfy it. In § 4 we consider that the "Norm" satisfying the triangular inequality actually characterizes the convex linear topological space.

1. Definitions. Kolmogoroff's Definition (Definition K). Let L be a linear Hausdorff space. If the vector operations $x+y$ and $t\cdot x$ are continuous with respect to this topology, then L is said to be a linear topological space.

Neumann's Definition (Definition N). Let L be a linear space. If L has family A of subsets U in L satisfying the following conditions, it is said to be a linear topological space, and is denoted by $L(A)$. A and U are said to be the neighbourhood system and neighbourhood, respectively.

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1) Kolmogoroff, Zur Normierbarkeit Eines Allgemeinen Topologischen Linear Raumes (Studia Math., Tom. V).

2) von Neumann, On complete Topological spaces (Trans. Amer. Math. Soc. XXXVII (1935)).

3) Hyers, Pseudo-normal Linear Space and Abelian Groups (Duke Math. Journ. Vol. 5 (1939)).

4) Wehausen, Transformations in Linear Topological space (Duke Math. Journ. Vol. 4 (1938)).