NOTES ON BANACH SPACE (XI): BANACH LATTICES WITH POSITIVE BASES*)

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After G. Birkhoff [2], in the Banach lattices the lattice operations are uniformly cotinuous with respect to the strong (norm) topology. This theorem does not hold for the weak topology. For, let $x_n(t)$ $(n=1,2,\ldots)$ be the sequence of Rademacher functions, considering in the Banach lattice L^p $(p \geqq 1)$, its positive part converges weakly to 1/2, however $x_n(t)$ converges weakly to zero. The first part of this paper is devoted to investigate Banach lattices in which the lattice operations are weakly continuous. This is given by the character of the intervals. In the second part, we give characteristic properties of the k-space, whose definition is given in Definition 3.

Throughout this paper, we shall use the technical terms and notations in Birkhoff's book [2] without any explanation.

1. We will prove firstly the following

Theorem 1. *The lattice operations of the conjugate space of a separable Banach lattice are continuous with respect to the weak topology as functionals if and only if the interval of the Banach lattice is strongly compact.*

Proof: Suppose that the lattice operations are weakly continuous in the conjugate space E^* of a separable Banach lattice E. Then

$$f_n^+(x) = \sup\{f(y)\,;\, 0 \leqq y \leqq x\}$$

converges to zero whenever f_n converges weakly to zero. Hence it holds $|f_n(y)| \leqq f_n^+(y) - f_n^-(y) < \varepsilon$ for any positive number ε, sufficiently large n and y with $0 \leqq y \leqq x$. That is, $\{f_n(y)\}$ converges uniformly on the interval $(0, x)$, and so the interval is strongly compact by a compactness theorem due to I. Gelfand [5] and R. S. Phillips [9].

Conversely, if each interval $(0, x)$ is compact in the strong topology, then $\{f_n(y)\}$ converges uniformly on it whenever f_n converges weakly to zero, whence $\sup_y f(y)$ converges to zero for any y belonging to the interval, and

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