

**ON BOREL'S DIRECTIONS OF MEROMORPHIC FUNCTIONS
OF FINITE ORDER^{*)}**

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1. Introduction.

Let $w(z)$ be meromorphic for $|z| < \infty$ and

$$T(r) = \int_0^r \frac{S(r)}{r} dr,$$

where

$$S(r) = \frac{1}{\pi} \int_0^r \int_0^{2\pi} \left(\frac{|w'(te^{i\theta})|}{1 + |w(te^{i\theta})|^2} \right)^2 t dt d\theta \quad (1)$$

be its Nevanlinna's characteristic function and

$$\overline{\lim}_{r \rightarrow \infty} \log T(r) / \log r = \rho \quad (2)$$

be its order. If $\rho < \infty$, then by Borel's theorem, for any $\varepsilon > 0$,

$$\sum_{\nu} 1/|z_{\nu}(a)|^{\rho+\varepsilon} < \infty$$

for any a and if $0 < \rho < \infty$,

$$\sum_{\nu} 1/|z_{\nu}(a)|^{\rho-\varepsilon} = \infty$$

for any a , with two possible exceptions, where $z_{\nu}(a)$ are zero points of $w(z) - a$.

Valiron¹⁾ proved that there exists a direction J , which is called a Borel's direction, such that

$$\sum_{\nu} 1/|z_{\nu}(a, \Delta)|^{\rho-\varepsilon} = \infty,$$

^{*)} Received October 1, 1949.

1) G. Valiron: Recherches sur le théorème de M. Borel dans la théorie des fonctions méromorphes. Acta Math. **52** (1928).