ON BOREL'S DIRECTIONS OF MEROMORPHIC FUNCTIONS OF FINITE ORDER*)

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1. Introduction.

Let $w(z)$ be meromorphic for $|z| < \infty$ and

$$T(r) = \int_0^r \frac{S(r)}{r}\, dr,$$

where

$$S(r) = \frac{1}{\pi} \int_0^r \int_0^{2\pi} \left(\frac{|w'(te^{i\theta})|}{1 + |w(te^{i\theta})|^2} \right)^2 t\, dt\, d\theta \tag{1}$$

be its Nevanlinna's characteristic function and

$$\varlimsup_{r \to \infty} \log T(r)/\log r = \rho \tag{2}$$

be its order. If $\rho < \infty$, then by Borel's theorem, for any $\varepsilon > 0$,

$$\sum_\nu 1/|z_\nu(a)|^{\rho+\varepsilon} < \infty$$

for any a and if $0 < \rho < \infty$,

$$\sum_\nu 1/|z_\nu(a)|^{\rho-\varepsilon} = \infty$$

for any a, with two possible exceptions, where $z_\nu(a)$ are zero points of $w(z) - a$.

Varilon[1] proved that there exists a direction J, which is called a Borel's direction, such that

$$\sum_\nu 1/|z_\nu(a, \Delta)|^{\rho-\varepsilon} = \infty,$$

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1) G. Valiron: Recherches sur le théorème de M. Borel dans la théorie des fonctions méromorphes. Acta Math. **52** (1928).