ON THE MAXIMAL HILBERT ALGEBRAS

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H. Nakano [4][1] has extended the results of W. Ambrose concerning his "proper H-algebras" (see [1]), by introducing the notion of "Hilbert algebras". In his paper, he showed, among others, that, to each Hilbert algebra, there exists a distinguished extension of it (*maximal extension*) which cannot be extended properly in any way ([4; Theorem 2.2]). After he told to the author this result, W. Ambrose's second paper [2] concering "H-systems" has appeared. Considering the inner relations of these notions, the author was able to show that every Hilbert algebra can be extended uniquely to a maximal one, and the considerations of maximal Hilbert algebras and H-systems are the same thing, *i. e.* the "bounded algebra" of an H-system is no other than our maximal Hilbert algebra. The structure of this algebra was also determined completely in some extent (*i. e.*, except that we have to introduce the separability assumption at a certain point) by the use of the F. J. Murray and J. von Neumann's theory on rings of operators.

In this paper we shall concern with the existence and unicity of a given Hilbert algebra and also some principal properties of the maximal Hilbert algebras deduced from it. As to the structure, we shall only give the results, as the proof is considerably long though the method is not so new. The fundamentals for the proof will be mentioned. The notions and notations in [4; § 1] will be used freely.

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1. The maximal extension of a Hilbert algebra.

Let a Hilbert algebra $\mathfrak{A}$ in $\mathfrak{H}$ be given.

DEFINITION 1. A Hilbert algebra $\widetilde{\mathfrak{A}}$ in $\mathfrak{H}$ is called the ***extension*** of $\mathfrak{A}$ if $\mathfrak{A}$ is contained in $\widetilde{\mathfrak{A}}$, and the multiplication and the adjoint operation in $\mathfrak{A}$ are preserved in $\widetilde{\mathfrak{A}}$. If there exists no proper extension of $\mathfrak{A}$ we call the Hilbert algebra $\mathfrak{A}$ ***maximal.*** If $\widetilde{\mathfrak{A}}$ is a maximal Hilbert algebra and at the same time the extension of $\mathfrak{A}$ we call it the ***maximal extension*** of $\mathfrak{A}$.

We shall state the existence of the maximal extension of the given Hilbert algebra $\mathfrak{A}$ and deduce some fundamental properties of it.

For this purpose, let us consider first the nature of the extension $\widetilde{\mathfrak{A}}$ of an $\mathfrak{A}$. As $\mathfrak{A}$ is contained in $\widetilde{\mathfrak{A}}$, there correspond to an arbitrary $a \in \mathfrak{A}$ two

1) Numbers in brackets denote the number of literature at the end of the paper.