THE PARACOMPACTNESS OF CW-COMPLEXES

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The category of paracompact normal spaces is a sufficiently broad class in which some methods of algebraic topology may be applied. It is well-known that this category contains all metric spaces, compact normal spaces and fully normal Hausdorff spaces. Simplicial complexes with the weak topology and CW-complexes in the sense of J. H. C. Whitehead [5, § 5][1] especially play important roles in the algebraic topology. Author [3, Lemma 4], J. Dugundji [2, Theorem 4] and D. G. Bourgin [1, Theorem 3] have been proved that any simplicial complex with the weak topology is paracompact. The purpose of the present note is to prove that any CW-complex is paracompact.

1. Notations. Let X be a space and W be a subset of X and $\mathfrak{V} = \{V_j\}$ be a family of subsets $V_j \subset X$ and $f: Y \to X$ be a continuous map of a space Y into X. We shall use the following notations·

$$\overline{\mathfrak{V}} = \{\overline{V}_j\} \quad (\overline{V}_j \text{ denotes the closure of } V_j \text{ in } X),$$

$$\mathfrak{V} \cap W = \{V_j \cap W\},\ \mathrm{St}(W ; \mathfrak{V}) = \cup V_j(V_j \cap W \neq \phi,\ V_j \in \mathfrak{V}),$$

$$\mathrm{St}(\mathfrak{V}) = \{\mathrm{St}(V_j ; \mathfrak{V}) | V_j \in \mathfrak{V}\},\ f^{-1}(\mathfrak{V}) = \{f^{-1}(V_j) | V_j \in \mathfrak{V}\}.$$

And $\mathfrak{U} > \mathfrak{V}$ means that each element of $\mathfrak{U}$ is contained in some element of $\mathfrak{V}$.

Let E^n be the subset of n-dimensional Euclidean space defined by

$$-1 \leqq x_i \leqq 1 \qquad (i = 1, \dots, n).$$

The boundary of E^n is denoted by S^{n-1}. For a point $x \in S^{n-1}$ and a real number $t (0 \leqq t \leqq 1)$ let $(x,\ t)$ denotes a point which divides the segment joining x to the center $0 = (0, \cdots, 0)$ of E^n in the ratio $t : 1 - t$.

For a given point $(x_0\ t_0)$ $(x_0 \in S^{n-1},\ 0 < t_0 < 1)$ let V be an open set of S^{n-1} which contains x_0 and let ε be a number such that $0 < \varepsilon < \min(t_0,\ 1 - t_0)$. Then the open set $W(x_0,\ t_0) = \{(x,\ t) | x \in V, |t - t_0| < \varepsilon\}$ is called a regular open set of E^n with the center $(x_0,\ t_0)$ and the bottom $\mathrm{B}[W(x_0,\ t_0)] = V$ and the breadth $\delta[W(x_0,\ t_0)] = \varepsilon$.

2. Two lemmas. Here we shall prove two lemmas which are used in the proof of our main theorem.

LEMMA 1. *Let P be a union*[2] *of at most $(n+1)$-dimensional element E^r_λ which are mutually disjoint. Let $\mathfrak{U} = \{U_\alpha\}$ be any open covering of P. Let*

1) Numbers in brackets refer to the references cited at the end of this note.

2) P is topologized so that each E^r_λ with its own topology is both open and closed in P.