ANALYTIC FUNCTIONS STAR-LIKE OF ORDER $p$
IN ONE DIRECTION

TOSHIO UMEZAWA

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1. Introduction. Recently A. W. Goodman and M. S. Robertson [1] have studied typically-real functions of order $p$ which were defined as follows:

A function

\[ f(z) = \sum_{n=0}^{\infty} b_n z^n \]  

is said to be a member of the class $T(p)$, if in (1.1) the coefficients $b_n$ are all real and if either (I) $f(z)$ is regular in $|z| \leq 1$ and $3f(e^{i\theta})$ changes sign $2p$ times as $z = e^{i\theta}$ traverses the boundary of the unit circle, or (II) $f(z)$ is regular in $|z| < 1$ and if there is a $\rho < 1$ such that for each $r \in \rho < r < 1$, $3f(re^{i\theta})$ changes sign $2p$ times as $z = re^{i\theta}$ traverses the circle $|z| = r$.

Concerning the above class of functions A. W. Goodman [2] has obtained the following result:

Let

\[ f(z) = 2^{q} + \sum_{n=q+1}^{\infty} b_n z^n \]

be a function of the set $T(p)$. Suppose that in addition to the $q$-th order zero at $z = 0$, the function $f(z)$ has exactly $s$ zeros, $\beta_1, \beta_2, \ldots, \beta_s$, such that $0 < |\beta_j| < 1$, $j = 1, 2, \ldots, s$. Finally let the non-negative integer $t$ be defined by

\[ q + s + t = p \geq 1 \]

and let $m = [(t + 1)/2]$. Then

\[ |b_n| \leq B_n, \quad n = q + 1, q + 2, \ldots, \]

where $B_n$ is defined by

\[ F(z) = z^q \frac{z^q}{(1-z)^{q+1}} \left( \frac{1+z}{1-z} \right)^{2m} \prod_{j=1}^{s} \left( 1 + \frac{z}{|\beta_j|} \right) (1 + z|\beta_j|) \]

\[ = z^t + \sum_{n=q+1}^{\infty} B_n z^n. \]

When $t$ is odd or when $t = 0$, $F(z) \in T(p)$ and the inequality (1.4) is sharp.

Now in the present paper we shall introduce wider classes of functions, to be defined precisely in §2, whose coefficients are not necessarily real, proving that inequalities similar to (1.4) can be obtained.

For the special case when $t = 0$ in (1.3) the above work has already