CONTRIBUTIONS TO THE THEORY OF FUNCTIONS OF A BICOMPLEX VARIABLE[1)]

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INTRODUCTION

1. The customary axiomatic definition of the system of ordinary complex numbers may be given as follows: (see Dickson, [1][2)])

Let a, b, c, d be real numbers. Two couples (a,b) and (c,d) are called equal if and only if $a = c$ and $b = d$. Addition and multiplication of two couples are defined by the formulas:

$$(a,b) + (c,d) = (a+c, b+d)$$
$$(a,b)\,(c,d) = (ac-bd,\ ad+bc).$$

Addition and multiplication are commutative and associative, and the distributive law holds.

Subtraction is defined as the operation inverse to addition. It is always possible and unique.

Division is defined as the operation inverse to multiplication. Division, except by $(0,0)$ is possible and unique:

$$\frac{(c,d)}{(a,b)} = \left(\frac{ac+bd}{a^2+b^2},\ \frac{ad-bc}{a^2+b^2}\right).$$

Now let $(a,0)$ be a, and $(0,1)$ be i. Then

$$i^2 = (0,1)\,(0,1) = (-1,0) = -1$$
$$(a,b) = (a,0) + (0,b) = (a,0) + (b,0)\,(0,1) = a+bi.$$

Thus the set of all real couples, with the above definitions, becomes the field of all complex numbers. The theory of complex-valued analytic functions of a complex variable has been extensively developed.

2. The question next arises as to what occurs if the above definitions are applied to couples of complex numbers, and the corresponding function theory investigated. This new system permits the same definition of the four fundamental operations, except that division will not be possible by the couple (a,b)

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2) Numbers in brackets refer to the bibliography at the end of the paper.