COINCIDENCE POINTS OF A CURVE

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1. Halphen [1] defined a coincidence point of a plane curve in the following way. There is a pencil of cubics having 8-point contact with the curve and with each other at a given point P. These cubics all pass through a further point H If H coincides with P, then P is called a *coincidence point* of the curve. The cubics therefore have 9-point contact with each other at P, but only 8-point contact with the curve. Halphen [2] also shows that a necessary and sufficient condition that a simple point P is a coincidence point is that there is a cubic with a double point at P such that P counts nine times as an intersection of the curve and the cubic: in general, such a cubic will have a node at P, one branch having 8-point contact with the curve.

The object of this note is to determine the number of coincidence points of a curve in terms of its Plücker numbers. We shall assume that the curve has no singularities other than nodes or cusps, and we shall not regard a cusp or an inflexion as a coincidence point.

2. We first consider the case where the curve has no cusps. As a preliminary result, we find the number of simple points P such that a cubic through 7 fixed points of the curve, $O_1, \ldots . O_7$, has a node at P. The cubics through $O_1, \ldots . O_7$ form a net, and the nodes lie on the Jacobian of the net, which is of order 6 and has nodes at $O_1, \ldots, O_7$ [3]. Hence the number of points is $6n - 14$.

We now consider the (a, a') correspondence between a point P of the curve and a point Q where the cubic through 6 fixed points of the curve, $O_1, \ldots, O_6$, and having a node at P meets the curve again. If γ is the valency of the correspondence, then we have $a = 3n - 8$, $a' = 6n - 14$ (from the last paragraph), $\gamma = 2$. By the Cayley-Brill formula [4], the number of coincidences is $9n - 22 + 4p$. This is therefore the number of points P such that there is a cubic through $O_1, \ldots, O_6$ which has a node at P, one branch having 2-point contact at P.

We now consider the (a, a') correspondence between a point P of the curve and a point Q where the cubic through 5 fixed points of the curve, $O_1, \ldots, O_5$ and having a node at P, one branch having 2-point contact, meets the curve again. We now have $a = 3n - 8$, $a' = 9n - 22 + 4p$, $\gamma = 3$. Hence the number of coincidences is $12n - 30 + 10p$. This is therefore the number of points P such that there is a cubic through $O_1, \ldots, O_5$ which has a node at P, one branch having 3-point contact at P.

Proceeding in this way, we arrive at the result that the number of points P such that there is a cubic with a node at P, one branch having 8-point contact at P is $27n - 70 + 70p$. This would include the inflexions,