REMARKS ON PREDICTION PROBLEM IN THE THEORY OF STATIONARY STOCHASTIC PROCESSES

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1. Suppose that $X(t)$ is a continuous stationary process in wide sense, $E\{X(t)\} = 0, E\{|X(t)|^2\} < \infty$ and $\rho(u)$ is the correlation function $E\{X(t+u)\overline{X(t)}\}$ which is represented as

$$(1.1) \qquad \rho(u) = \int_{-\infty}^{\infty} e^{iux} dF(x),$$

$F(x)$ being a bounded, non-decreasing function.

In previous papers [1], [2], we have discussed about Wiener's prediction theory. The object of the present paper is to give some remarks on prediction problem in the case where $F(x)$ satisfies a further condition that

$$(1.2) \qquad \int_{-\infty}^{\infty} x^{2p} dF(x) < \infty,$$

p being a positive integer.

We shall first give some definitions, notations and some known results.

Let $K(\theta)$ be a function of bounded variation in every finite interval in $[0, \infty)$. If $\int_0^A e^{-ix\theta} dK(\theta)$ converges in $L_2(-\infty, \infty)$ with respect to $F(x)$ to a function $k(x)$ when $A \to \infty$, $K(\theta)$ is called to belong to $\mathbf{K}(0, \infty)$. That is, if

$$(1.3) \qquad \lim_{A\to\infty} \int_{-\infty}^{\infty} \left| \int_0^A e^{-ix\theta} dK(\theta) - k(x) \right|^2 dF(x) = 0,$$

then $K(\theta) \in \mathbf{K}(0, \infty)$ and this fact is denoted as

$$(1.4) \qquad \underset{A\to\infty}{\text{l.i.m.}}\ L_2(F) \int_0^A e^{ix\theta} dK(\theta) = k(x),$$

and $k(x)$ is called the Fourier-Stieltjes transform of $K(\theta)$ in $L_2(F)$.

It is known[3] that if $K(\theta) \in \mathbf{K}(0, \infty)$, then

$$(1.5) \qquad \underset{A\to\infty}{\text{l.i.m.}} \int_0^A X(t-\theta) dK(\theta)$$

exists. l.i.m. means the limit in variance. (1.5) is denoted as

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