A NOTE ON EILENBERG-MACLANE INVARIANT

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Introduction. It was proved in [3] that if X is arcwise connected and $\pi_i(X) = 0$ for $i < n, n < i < q$, then $H_i(X, G) \cong H_i(K, G)$ for $i < q$, and $H_q(X, G)/\Sigma_q(X, G) \cong H_q(K, G)$, where $K = K(\pi_m(X, m))$, and $\Sigma_q(X, G)$ is the spherical subgroup of the q-th homology group $H_q(X, G)$. In other words under the above conditions, the group π_n determines in a purely algebraic fashion the homology structure of X in dimension $< q$. The group π_n also partially determines the q-dimensional homology group of X. In [3] Eilenberg-MacLane invariant $\mathbf{k}^{q+1}$ determines fully the structure of X in the dimension $\leqq q$.

A. L. Blakers introduced the notions of group system and set system in [2]. It was proved that if in the set system $\mathfrak{S} = \{X_i\}$ the natural homomorphisms $\pi_i(X_{i-1}) \to \pi_i(X_i)$ for all $i < q$ $(q > 0)$ are trivial, then the chain transformation κ induces isomorphism $\kappa_* : H_i(S(\mathfrak{S})) \cong H_i(K(\Pi(\mathfrak{S}))$ for all $i < q$, and for $i = q$, the induced homomorphism $\kappa_* : H_q(K(\mathfrak{S})) \to H_q(K(\Pi(\mathfrak{S}))$ is onto.

In § 2 we give a generalization of Eilenberg-MacLane invariant $\mathbf{k}^{q+1}(\Phi)$; this invariant is a cohomology class of a suitable algebraic cohomology group $H^{q+1}(K(\Pi(\mathfrak{S}), \pi_q(X_q))$ of the group $K(\Pi(\mathfrak{S}))$, with coefficients in $\pi_q(X_q)$.

It is shown that this invariant $\mathbf{k}^{q+1}(\Phi)$ fully determines the structure $S(\mathfrak{S})$ in the dimension $\leqq q$, and we have the following:

THEOREM. *If the natural homomorphisms $\pi_i(X_{i-1}) \to \pi_i(X_i)$ for $i < q, q > 0$ are trivial, then*

$$H^i(S(\mathfrak{S}), G) \cong H^i(K(\Pi(\mathfrak{S}), G) \quad \textit{for } i < q$$
$$H^q(S(\mathfrak{S}), G) \cong H^q(K^*, G),$$

where K^* is the new complex which we will define in § 3.

The main purpose of the present paper is to show the second part of the above theorem.

In § 4 we state algebraic considerations.

1. Preliminaries. We shall use notations and terminologies in [2] and [3].

Let X be an arcwise connected topological space with a point x_0 which will be used as base point for all of the homotopy groups considered in the sequel. Let a sequence $\mathfrak{S} = \{X_i\}$, $i = 0, 1, \ldots$ be a set system in X (cf. [2]). With the system we associate the groups $\pi_i(\mathfrak{S}) = \pi_i(X_i, X_{i-1}), i = 1, 2, \ldots$ with x_0 as base point. ($\pi_1(\mathfrak{S}) = \pi_1(X_1, X_0) = \pi_1(X)$.) We consider operator homomorphisms $\Delta_i : \pi_i(\mathfrak{S}) \to \pi_{i-1}(\mathfrak{S})$, for $i = 2, 3\ldots$

(1.1) *For each set system $\mathfrak{S}$, the groups $\pi_i(\mathfrak{S})$ and homomorphisms Δ_i form*