NOTES ON TAUBERIAN THEOREMS FOR RIEMANN SUMMABILITY

KENJI YANO

(Received February 1, 1956 and in revised forms November 12, 1956 and September 13, 1957)

1. **Introduction.** In a given series

$$\sum a_n = \sum_{n=1}^{\infty} a_n \qquad (a_0 = 0),$$

if

(1.1) $$\sum_{n=1}^{\infty} s_n \frac{\sin nt}{n} \qquad \left(\text{where } s_n = \sum_{\nu=0}^{n} a_\nu\right)$$

converges for every value of t $(0 < t < \delta \leqq 2\pi)$, and

$$\lim_{t \to +0} \frac{2}{\pi} \sum_{n=1}^{\infty} s_n \frac{\sin nt}{n} = s,$$

then we call that the series $\sum a_n$ is summable (R_1) to s, and write

$$\sum a_n = s \ (R_1).$$

Similarly, we call that the series $\sum a_n$ is summable $(R, 1)$ to s, and write $\sum a_n = s(R, 1)$, if the series

(1.2) $$\sum_{n=1}^{\infty} a_n \frac{\sin nt}{nt}$$

converges for every value of t $(0 < t < \delta \leqq 2\pi)$, and

$$\lim_{t \to +0} \sum_{n=1}^{\infty} a_n \frac{\sin nt}{nt} = s.$$

The summabilities (R_1) and $(R, 1)$ have been studied by many writers, O. Szász [4, 5, 6, 7], G. Sunouchi [1], H. Hirokawa and G. Sunouchi [3] and others. In this note we shall unify and extend the theorems due to the above writers. Generally speaking, the Riemann summabilities are near to the convergence of series and so Tauberian conditions of the above authors may be replaced by the conditions on s_n^{-s} $(s > 0)$, which will be defined in a moment.

We denote by s_n^γ the n-th Cesàro sum of order γ of the series $\sum a_n$, i. e.

$$s_n^\gamma = \sum_{\nu=0}^{n} A_{n-\nu}^\gamma a_\nu \qquad (a_0 = 0),$$

where A_n^γ is defined by the identity

$$(1 - x)^{-\gamma-1} = \sum_{n=0}^{\infty} A^\gamma x^n \qquad (|x| < 1),$$