THE HOLONOMY COVERING SPACE IN PRINCIPAL FIBRE BUNDLES

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1. Introduction. Recently L. Auslander and L. Markus [2][1] have studied on flat affinely connected manifolds. They have defined the holonomy covering space over the manifold and have shown that it has similar nature of a minimal universal covering space.

In this paper we shall extend these to principal fibre bundles with locally flat connections.

We shall first recall the definition of a (differentiable) principal fibre bundle [5, 7, 9, 10, 11] and denote it by $P(M, \pi, G)$, where P is the bundle space, M the base space, π a projection of P onto M and G a Lie group (not necessarily connected) acting on P on the right. By differentiability we shall always understand that of class C^∞. We shall give in $P(M, \pi, G)$ a connection Γ by the distribution $\Gamma : u \to \Gamma_u$ (horizontal subspace at u), where $u \in P$, or equivalently by a connection form ω in P with values in the Lie algebra $\mathfrak{g}$ of G [1, 3, 5, 7, 9, 10]. It is well known that the structure equation is given by

$$d\omega(\overline{u_1}, \overline{u_2}) = -\frac{1}{2}[\omega(\overline{u_1}), \omega(\overline{u_2})] + \Omega(\overline{u_1}, \overline{u_2}) \tag{1}$$

on P with connection Γ, where $\overline{u_1}$ and $\overline{u_2}$ are any vector fields on P, the bracket is the bracket operation in the Lie algebra $\mathfrak{g}$ and Ω is the curvature form of the connection Γ.

Let $P(M, \pi, G)$ be a principal fibre bundle and let h be a mapping of a manifold M' into M. Let $h^{-1}(P)$ be the set of points (x', u) of $M \times P$ such that $\pi(u) = h(x')$. $h^{-1}(P)$ is clearly a principal fibre bundle and we call it the principal fibre bundle induced by h. The mapping $\bar{h}$ of $h^{-1}(P)$ into P defined by

$$\bar{h}(x', u) = u,$$

commutes with the right translation by G. Hence $\bar{h}$ is a bundle map of $h^{-1}(P)$ into P.

Let $\bar{h}$ be a bundle map of a principal fibre bundle $P(M, \pi, G)$ into another principal fibre bundle $P'(M', \pi', G)$ and Γ' a connection in P'. Then there exists a connection Γ in P which is naturally induced from Γ'. The form ω on P which defines Γ is given by

$$\omega = \omega' \circ \delta \bar{h}.$$

We shall Γ the connection induced from Γ' by $\bar{h}$ and ω the form induced from

1) Numbers in brackets refer to the bibliography at the end of the paper.