ON CONVERGENCE CRITERIA FOR FOURIER SERIES II

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1. Introduction. Let $\varphi(t)$ be integrable in $(0, \pi)$, even, periodic of period 2π, and let

$$\varphi(t) \sim \frac{1}{2} a_0 + \sum_{n=1}^{\infty} a_n \cos nt.$$

We write

$$\Phi_\alpha(t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t-u)^{\alpha-1}[\varphi(u) - s]\, du \qquad (\alpha > 0),$$

and

$$s_n = \frac{1}{2} a_0 + \sum_{\nu=1}^{n} a_\nu.$$

F. T. Wang [1] has proved that if $\beta > \alpha > 0$, $\varphi(t) \in L$, $\Phi_\alpha(t) = o(t^\beta)$ as $t \to 0$ and if $a_n > -An^{-\alpha/\beta}$, $A > 0$, then $s_n \to s$ as $n \to \infty$. In order to prove this, Wang used the method of Riesz summability. In this paper we shall give an alternative proof by a method of generalized de la Vallée Poussin summability. In § 4 we shall refer to jump functions. This note is a continuation of K. Yano [5], but may be readed free from it.

DEFINITION 1. We define $g(x)$ such as
$1°$ $g(x) > 0$ for $x \geqq x_0 > 0$, $2°$ $g(x) \uparrow \infty$ as $x \uparrow \infty$, and $3°$ $H \leqq g(x^\delta)/g(x) \leqq 1$, $0 < \delta < 1$ for all $x \geqq x_0$, where $H = H(\delta)$ is a positive constant depending on δ only.

Then we see easily that $g(x) = o(x^\varepsilon)$ as $x \to \infty$ for every positive ε. In this definition we require no differentiability of $g(x)$. We may take for $g(x)$, e. g.,

$$\log x,\ (\log x)^\alpha \log\log x\ (\alpha \geqq 0) \text{ and } \log_p x,$$

where $\log_p$ denotes the p-times iterated logarithm. For the sake of simplicity we denote $(g(x))^\alpha$ by $g(x)^\alpha$ throughout this paper.

THEOREM 1.[1] *Let $\beta \geqq \alpha > 0$ and let $g(x)$ be unity or defined by Definition 1. If*

$$(1.1) \qquad \int_0^t |\Phi_\alpha(u)|\, du = o\left(t^{\beta+1}/g\left(\frac{1}{t}\right)\right) \qquad (t \to 0),$$

and if for any assigned positive ε

1) $\varphi(t)$ requires no integrability in Lebesgue sense.