THE DECOMPOSITION OF A DIFFERENTIABLE MANIFOLD AND ITS APPLICATIONS.

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Introduction. Since G. de Rham [2] proved an interesting theorem concerning the decomposition of simply-connected, complete, reducible Riemannian manifolds, it has been referred to by many authors. The author [3] already attempted an extension of the theorem to an affinely connected manifold. In this note let us treat further to extend it to a differentia le manifold. For this purpose, we shall first introduce the notion of a locally decomposed C^u-manifold with latticed maps (§ 1). In such a manifold, we prove a theorem on its fundamental group and we show that the manifold decomposes globally if it is simply-connected (Theorems 1, 2). Further, as its applications, we prove that locally decomposed, affinely connected manifold and Finsler manifold (§ 4) admit always latticed maps and we show that they decompose globally if they are simply-connected (Theorems 3, 4). All of these results are nothing but extensions of the G. de Rham's theorem, and further note that the idea is analogous to that in [3]. Throughout the whole discussion, let us suppose that the indices run as follows:

$$a,\ b,\ c = 1,\ 2, \ldots\ldots,\ r;\ i,\ j,\ k = r+1,\ r+2, \ldots\ldots,\ n;$$
$$\alpha,\ \beta,\ \gamma = 1,\ 2, \ldots\ldots,\ n.$$

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1. Locally decomposed C^u-manifolds with latticed maps. We take an n-dimensional connected manifold M of class C^u such that the maximal system Ω of admissible coordinate neighbourhoods has the following properties:

1) In each neighbourhood $U \in \Omega$, its coordinate system (x^α) consists of all of (x^α)'s such that $0 < x^\alpha < 1$.

2) For each pair $U, U' \in \Omega$, $U \cap U' \neq 0$, if x is any point of $U \cap U'$ there is a neighbourhood $V \subset U \cap U'$ of x where the transformation from the coordinates x^α in U to the ones x'^α in U' is expressed by decomposed relations

$$x'^a = x'^a(x^1, \ldots\ldots x^r), \quad x'^i = (x^{r+1}, \ldots\ldots, x^n)$$

(x'^a depends on x^a only and x'^i on x^i only).