ON THE SUMMATION OF MULTIPLE FOURIER SERIES[1]

KÔSI KANNO

(Received June 17, 1958)

1. Generalities. Let $f(x_1, \ldots\ldots, x_k) = f(x)$ be a real valued integrable function periodic with period 2π in $0 \leqq x_i \leqq 2\pi$, $i = 1, 2, \ldots\ldots k$. Following S. Bochner [1] and K.Chandrasekharan [2], we define the *'spherical means'* $f(x, t)$ of a function $f(x)$ at a point $x = (x_1 \ldots\ldots, x_k)$, for $t > 0$,

$$(1.\ 1)\qquad f(x, t) = \frac{\Gamma(k/2)}{2(\pi)^{k/2}} \int_\sigma f(x_1 + t\xi_1, \ldots\ldots, x_k + t\xi_k)\, d\sigma_\xi,$$

where σ is the sphere $\xi_1^2 + \ldots\ldots + \xi_k^2 = 1$ and $d\sigma_\xi$ is its $(k-1)$ – dimentional volume element. $f(x, t)$ considered as a function of the single variable t exists for almost all t, and integrable in every finite t-interval.

If $p > 0$, we define

$$(1.\ 2)\qquad f_p(x, t) = \frac{2}{B(p, k/2)\, t^{2p+k-2}} \int_0^t (t^2 - s^2)^{p-1} s^{k-1} f(x, s)\, ds,$$

which called the spherical mean of order p of the function $f(x)$. At a point x, we write $f_p(x, t) = f_p(t)$ for $p \geqq 0$, where we assume that $f_0(x, t) = f(x, t)$. The following properties of $f_p(t)$ are known [2].

$$(1.\ 3)\qquad \int_0^u t^{k-1} |f(x, t)|\, dt = O(u^k), \qquad \text{as } u \to \infty.$$

$$(1.\ 4)\qquad \int_0^u t^{k-1} |f(x, t)|\, dt = o(1), \qquad \text{as } u \to 0.$$

$$(1.\ 5)\qquad f_p(u) = O(1), \quad \text{for } p \geqq 1, \qquad \text{as } u \to \infty.$$

Further, if we define, for $p \geqq 0$ [2],

$$(1.\ 6)\qquad \varphi_p(t) = t^{2p+k-2} f_p(t)\, B(p, k/2)/2^p\Gamma(p).$$

then we have, for $p + q \geqq 1$,

$$(1.\ 7)\qquad \varphi_{p+q}(t) = \frac{1}{2^{q-1}\Gamma(q)} \int_0^t (t^2 - s^2)^{q-1} s\, \varphi_p(s)\, ds.$$

It is clear for (1. 7) that if $p \geqq 1$ then $\varphi_p(t)$ is absolutely continuous in every finite interval excluding the origin.

Next, let us write the Fourier series of $f(x)$ in the form,

1) The problem considered here was suggested by Professor G. Sunouchi.