ON THE RELATION BETWEEN HARMONIC SUMMABILITY AND SUMMABILITY BY RIESZ MEANS OF CERTAIN TYPE

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1. An infinite series $\sum_{n=0}^{\infty} u_n$ with partial sums $s_n = \sum_{0}^{n} u_k$ is said to be *summable by Harmonic means* [3], if the sequence $\{y_n\}$ tends to a limit as $n \to \infty$, where

$$(1.\ 1) \qquad y_n = \frac{b_n s_0 + b_{n-1} s_1 + \ldots\ldots + b_0 s_n}{b_0 + b_1 + \ldots\ldots + b_n}, \quad \left(b_n = \frac{1}{n+1}\right).$$

We write $B_n = b_0 + b_1 + \ldots\ldots + b_n$ so that $B_n \sim \log n$.

The main interest of the method lies in the Tauberian theorem associated with it.

THEOREM A [2]. *If Σu_n is summable by Harmonic means, and*

$$u_n = O(n^{-\delta}) \qquad 0 < \delta < 1,$$

then Σu_n is convergent.

If $\delta = 1$, Theorem A reduces to well known Tauber's first theorem, in view of the fact that Harmonic summability implies (C, δ) summability for every $\delta > 0$.

If $p_n \geqq 0$, $p_0 > 0$, $\Sigma p_n = \infty$, (so that $P_n = p_0 + p_1 + \ldots\ldots + p_n \to \infty$), and

$$(1.\ 2) \qquad t_n = \frac{p_0 s_0 + p_1 s_1 + \ldots\ldots + p_n s_n}{p_0 + p_1 + \ldots\ldots + p_n} \to s$$

as $n \to \infty$, then we say that $s_n \to s(R, p_n)$ [1, p. 57]. If we choose $P_n = \exp n^{\alpha}$ $(0 < \alpha < 1)$, then the Tauberian condition of Theorem A is also the Tauberian condition of (R, p_n) summability.

The object of this note is to give an indirect proof of Theorem A by proving the following theorem:

THEOREM I. *If an infinite series Σu_n is summable by Harmonic means to the sum s, then it is also summable (R, p_n) to the same sum, where $P_n = \exp n^{\alpha}$ $(0 < \alpha < 1)$.*