AN ASPECT OF LOCAL PROPERTY OF $|R, \log n, 1|$ SUMMABILITY OF FOURIER SERIES

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1. 1. DEFINITION. Let S_n denote the n-th partial sum of the series Σa_n. We write

$$R_n = \left\{ S_1 + \frac{1}{2} S_2 + \ldots\ldots + \frac{1}{n} S_n \right\} \Big/ \log n.$$

Then the series Σa_n is said to be *absolutely summable* $(R, \log n, 1)$ or *summable* $|R, \log n, 1|$ if the sequence $\{R_n\}$ is of bounded variation, that is to say, the infinite series

$$\sum |R_n - R_{n+1}|$$

is convergent.

It has been pointed out by Bosanquet* that for the case $\lambda_n = \log n$, this definition is equivalent to the definition of the summability $|R, \lambda_n, 1|$ used by Mohanty [5], λ_n being a monotonic increasing sequence tending to infinity with n.

1. 2. Let $f(t)$ be a periodic function with period 2π and integrable (L) over $(-\pi, \pi)$. Without any loss of generality the constant term in the Fourier series of $f(t)$ can be taken to be zero, so that

(1. 2. 1) $$f(t) \sim \Sigma (a_n \cos nt + b_n \sin nt) = \Sigma A_n(t),$$

and

(1. 2. 2) $$\int_{-\pi}^{\pi} f(t)\, dt = 0.$$

We write

$$\varphi(t) = \frac{1}{2} \{f(x+t) + f(x-t)\}.$$

1. 3. It has been proved independently by Izumi [3] and Mohanty [5] that summability $|R, \log n, 1|$ of a Fourier series is not a local property of the generating function. The question, naturally arises as to what conditions

* L. S. Bosanquet, Mathematical Review, 12 (1951), 254, see review of the paper of Izumi [3].