ON THE RIESZ SUMMABILITY OF FOURIER SERIES

GEN-ICHIRÔ SUNOUCHI

(Received April 13, 1959)

Let $f(x)$ be an integrable and periodic function with period 2π, and let

$$(1) \qquad f(x) \sim \frac{1}{2}a_0 + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx).$$

F.T. Wang [4] proved the following theorem:
If $1 < \alpha < 2$, and the series

$$\sum_{n=2}^{\infty}(a_n^2 + b_n^2)(\log n)^{\alpha-1}$$

converges, then the Fourier series (1) is summable $(R, \exp(\log n)^\alpha, \delta)$ almost everywhere, for any positve δ.

In this note we shall give some better results than the above theorem.

THEOREM 1. *If* $1 < \alpha < \infty$, *and the series*

$$\sum_{n=2}^{\infty}(a_n^2 + b_n^2)\{\log(\log n)\}$$

converges, then Fourier series (1) *is summable* $(R, \exp(\log n)^\alpha, \delta)$ *almost everywhere for any positive* δ.

THEOREM 2. *If* $0 < \alpha \leqq 1$, *and the series*

$$\sum_{n=2}^{\infty}(a_n^2 + b_n^2)(\log n)^\alpha$$

converges, then the Fourier series (1) *is summable* $(R, \exp\{\exp(\log n)^\alpha\}, \delta)$, *almost everywhere for any positive* δ.

In Theorem 2, if we put $\alpha = 1$, then the convergency of $\sum_{n=2}^{\infty}(a_n^2 + b_n^2)\log n$ implies the (R, e^n, δ) summability of (1) almost everywhere. Since (R, e^n, δ) summation is equivalent to convergence, this case is nothing but the theorem of Kolmogoroff-Seliverstoff-Plessner. Thus our theorems link the theorem of Kolmogoroff-Seliverstoff-Plessner and the theorem of Fejér-Lebesgue. Improvement of our results may be difficult.

Our theorems are easy consequences of the following two propositions.

PROPOSITION 1. *The Lebesgue constant of* $(R, \Lambda_n, 1)$ *summation of the*