CERTAIN TYPES OF GROUPS OF AUTOMORPHISMS OF A FACTOR

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In studying the crossed products of rings of operators, we have shown in [4] that an arbitrary countable group admits a faithful representation as a group of outer automorphisms of an approximately finite factor on a separable Hilbert space (the so-called outer automorphic representation). The object of the present paper is to discuss the algebraic properties of groups of outer automorphisms of the approximately finite factor, which are the outer automorphic representations of certain torsion free groups.

An automorphism (a group of automorphisms) of a factor is said to be *ergodic* if it leaves only the center elementwise invariant. Our first task is to ask whether there exists an ergodic group of automorphisms of the approximately finite factor. Indeed, we shall find the necessary and sufficient condition that the outer automorphic representation of a group be ergodic. The second one is to examine the outer automorphic representation of a free product of two arbitrary torsion free countable groups and show that the crossed products of the approximately finite factor by such groups of automorphisms are not approximately finite.

1. In the starting point, we shall recall the construction in [4] of the outer automorphic representation of a countably infinite group G. Let Δ be a set of all functions α on G taking only the values 0, 1 such that $\alpha(g) = 0$ except for a finite number of g's. Then Δ is an additive group with the addition $[\alpha + \beta](g) = \alpha(g) + \beta(g)$ (mod 2). Further, define Δ' as a set of all functions φ on Δ taking only the values 0, 1 such that $\varphi(\gamma) = 0$ except for a finite number of γ's, and make Δ' into a group by the addition $[\varphi + \psi](\gamma) = \varphi(\gamma) + \psi(\gamma)$ (mod 2). Now we make the pair $\mathcal{G} = (\Delta', \Delta)$ into a group by defining

$$(\varphi, \alpha)(\psi, \beta) = (\varphi^{\beta} + \psi, \alpha + \beta)$$

where $\varphi, \psi \in \Delta', \alpha, \beta \in \Delta$ and $\varphi^{\beta}(\gamma) = \varphi(\gamma + \beta)$. Let $\{V_{(\varphi,\alpha)}\}_{(\varphi,\alpha)\in\mathcal{G}}$ be unitary operators on $l_2(\mathcal{G})$ defined by $[V_{(\varphi,\alpha)}f]((\psi,\beta)) = f((\psi,\beta)(\varphi,\alpha))$ for all $f \in l_2(\mathcal{G})$, then a factor $\mathbf{M}$ generated by $\{V_{(\varphi,\alpha)}\}_{(\varphi,\alpha)\in\mathcal{G}}$ is an approximately finite factor on a separadle Hilbert space. Hereupon, for each $g \in G$, define an automorphism T_g on Δ by