RIEMANN-CESÀRO METHODS OF SUMMABILITY IV

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1. Introduction. Let $a(u)$ be bounded mesurable in every finite interval of $u \geqq 0$ and let

$$s_\alpha(u) = \frac{1}{\Gamma(\alpha+1)}\int_0^u (u-x)^\alpha a(x)\,dx,\ \alpha > -1,$$
$$\sigma_\alpha(u) = \Gamma(\alpha+1)s_\alpha(u)/u^\alpha,\ \alpha > -1,$$

and let

$$\sigma_{-1}(u) = us_{-1}(u) = ua(u).$$

If $\sigma_\alpha(u) \to s$ as $u \to \infty$, then we say that the integral

$$\int_0^\infty a(u)du \tag{1.1}$$

is evaluable (C, α), $\alpha > -1$, to s and write

$$(C, \alpha)\int_0^\infty a(u)du = s.$$

If $\int_0^\infty |d\sigma_\alpha(u)|$ is finite and $\sigma_\alpha(u) \to s$ as $u \to \infty$, then we say that the integral (1. 1) is evaluable $|C, \alpha,|$ $\alpha > -1$, to s and write

$$|C, \alpha|\int_0^\infty a(u)du = s.$$

Recently, Rajagopal [5] defined the Riemann-Cesàro methods of summability for integrals. In the following, let p be a positive integer and let α be a real number such that $\alpha \geqq -1$. The integral (1. 1) is said to be evaluable (R, p, α) to s if the integral

$$C_{p,\alpha}^{-1} t^{\alpha+1}\int_0^\infty s_\alpha(u)\left(\frac{\sin ut}{ut}\right)^p du, \tag{1.2}$$

where

$$C_{p,\alpha} = \begin{cases} \dfrac{1}{\Gamma(\alpha+1)}\displaystyle\int_0^\infty u^{\alpha-p}\sin^p u\,du, & -1 < \alpha < p-1 \text{ or } \alpha = 0,\ p = 1, \\ 1, & \alpha = -1, \end{cases}$$

converges in some interval $0 < t < t_0$ and its limit tends to s as $t \to 0+$. The purpose of this paper is to establish the summability theorems for inte-