ON ALMOST-ANALYTIC VECTORS IN ALMOST-KÄHLERIAN MANIFOLDS[1]

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In pseudo-Kählerian manifolds, many interesting results concerning contravariant or covariant pseudo-analytic vectors are known.[2] Even though there were many papers about pseudo-Kählerian manifolds, but were few about almost-Kählerian ones. Recently, M. Apte generalized Bochner's theorem to compact almost-Kählerian manifolds. His work seems to be very suggestive for me. In the present paper we shall generalize several theorems in pseudo-Kählerian manifolds to almost-Kählerian ones. The main results are integral formulas on vector fields in compact almost-Kählerian manifolds.

In §1 and §2 we shall prepare identities and lemmas and in §3 and §4 define almost-analytic vectors which are generalizations of pseudo-analytic vectors. As applications of integral formulas in §5, we shall obtain several theorems in §6. In §7, we shall give a decomposition theorem of the Lie algebra of contravariant almost-analytic vectors in a compact almost-Kähler-Einstein manifold. The canonical connection will be introduced in §8 and in the last section, to contravariant almost-analytic vectors, we shall generalize Apte's theorem.

1. Identities. In an n-dimensional real differentiable manifold M with local coordiantes $\{x^i\}$, a tensor field $\varphi_j^{\ i}$ such that

$$\varphi_r^{\ i}\varphi_j^{\ r} = -\delta_j^{\ i} \tag{1.1}$$

is called an almost-complex structure. If an almost-complex structure $\varphi_j^{\ i}$ and a positive definite Riemannian metric tensor g_{ji} on M satisfy the relation

$$g_{rs}\varphi_j^{\ r}\varphi_i^{\ s} = g_{ji}, \tag{1.2}$$

then the pair $(\varphi_j^{\ i}, g_{ji})$ is called an almost-Hermitian structure. Then, from (1.1) and (1.2), we get

$$\varphi_{ji} = -\varphi_{ij}, \tag{1.3}$$

where $\varphi_{ji}=\varphi_j^{\ r}g_{ri}$. To an almost-Hermitian structure $(\varphi_j^{\ i}, g_{ji})$, an exterior dif-

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2) For example, cf. Yano, K. [7], Lichnerowicz, A. [3], Sasaki, S. and K. Yano [5], Yano, K. and I. Mogi [9].