# SEMI-GROUPS OF OPERATORS IN FRÉCHET SPACE AND APPLICATIONS TO PARTIAL DIFFERENTIAL EQUATIONS

ISAO MIYADERA

(Received December 16, 1958)

**1. Introduction.** This paper is concerned with semi-groups of operators in Fréchet space and its application to the Cauchy problem for some linear partial differential equations with constant coefficients.

A topological vector space is called a Fréchet space if it is locally convex, complete and metrizable.

We shall deal with a semi-group of operators $\{T(\xi); 0 \leqq \xi < \infty\}$ satisfying the following conditions:

(1) For each $\xi \geqq 0$, $T(\xi)$ is a continuous linear operator from a Fréchet space $X$ into itself and

$$T(\xi + \eta) = T(\xi)T(\eta) \qquad \text{for } \xi, \eta \geqq 0,$$
$$T(0) = I \text{ (the identity).}$$

(2) There exists a non-negative number $\sigma$ such that

$$\{e^{-\sigma\xi}T(\xi)x\,; \xi \geqq 0\}$$

is bounded in $X$ for each $x \in X$.

(3) $$\lim_{\xi \downarrow 0} T(\xi)x = x \qquad \text{for each } x \in X.$$

Since a Banach space is obviously a Fréchet space, our semi-groups are an extention of semi-groups of class $(C_0)$ in Banach space. (For semi-groups in Banach space see the book of E. Hille and R. S. Phillips [3].)

We first remark that the conditions (1) and (3) imply the condition (2) if $X$ is a Banach space. For $M \equiv \sup_{0 \leqq \xi \leqq 1} \|T(\xi)\| < \infty$ by the uniform boundedness theorem, and hence $\|T(\xi)\| \leqq M \cdot \exp(\xi \log M)$ for each $\xi \geqq 0$. But this is not true in general if $X$ is a Fréchet space.

EXAMPLE. We consider real valued functions of one real variable. $C^\infty$ denotes the space of $\infty$ times continuously differentiable functions. It is well known that the space $C^\infty$ becomes a Fréchet space under the family of semi-norms $\{p_{m,k}(\cdot)\,; m, k = 0, 1, 2, \cdots\cdots\}$, where

(1.1) $$p_{m,k}(x) = \sup_{|t| \leqq k} |x^{(m)}(t)| \qquad \text{for each } x \in C^\infty.$$

We define