TENSOR PRODUCTS OF COMMUTATIVE BANACH ALGEBRAS

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(Received November 5, 1959)

In [2] and [5], A. Hausner and G. P. Johnson announced the following theorem: Let G be a locally compact abelian group and X a complex commutative Banach algebra. Then the space of all regular maximal ideals topologized in the Gelfand sense of $L^1(G, X)$, the space of all X-valued Bohner integrable functions on G, is homeomorphic with the Cartesian product of $\mathfrak{M}(X)$ and $\hat{G}$ in the product topology, where $\mathfrak{M}(X)$ means the space of all regular maximal ideals and $\hat{G}$ the dual group of G. Moreover improving the result by B.Yood [10], A.Hausner has proved that the analogous result also holds valid for the space of all vector-valued continuous functions on a compact Hausdorff space [4]: that is for a commutative Banach algebra X and a compact Hausdorff space Ω, the space of all regular maximal ideals in $C(\Omega, X)$, the space of all X-valued continuous functions on Ω, is homeomorphic with the Cartesian product of $\mathfrak{M}(X)$ and Ω in the product topology.

Now it is known by Grothendieck [1] that $L^1(G, X)$ is isometric-isomorphic to $L^1(G) \hat{\otimes}_{\gamma} X$, the tensor product of $L^1(G)$ and X with γ-norm and $C(\Omega, X)$ to $C(\Omega) \hat{\otimes}_{\lambda} X$, the tensor product of $C(\Omega)$ and X with λ-norm. This fact suggests us that the tensor productorial treatment of the problem may be fruitful and make the background of their discussions clear.

Our first result is the following simultaneous generalization of the theorems by Hausner-Johnson and Yood-Hausner: Let A and B be commutative Banach algebras and suppose that $A \hat{\otimes}_{\alpha} B$, the tensor product of A and B for the cross norm α not less than λ-norm, is a Banach algebra, then the space of all regular maximal ideals in $A \hat{\otimes}_{\alpha} B$ topologized with the usual weak topology is homeomorphic with the product space of $\mathfrak{M}(A)$ and $\mathfrak{M}(B)$.

Next are shown some theorems about the regularity and semi-simplicity of the tensor product of A and B, which are also the generalizations of the results announced also in [2] and [5] in the case of $A = L^1(G)$ where G is a locally compact abelian group and the treated cross norm is γ-norm. In our arguments, the following family of linear mappings from $A \hat{\otimes}_{\alpha} B$ to A(or B) plays an