# REMARKS ON THE REALIZABILITY OF WHITEHEAD PRODUCT

(Received September 25, 1959)

KÔICHI IWATA AND HIROSHI MIYAZAKI

**1.** A. H. Copeland [2][1] investigated the problem of finding an $H$-structure of a $CW$-complex with two non-trivial homotopy groups. In the course of his study, an interesting result is obtained which combine the Eilenberg-MacLane invariant and the Whitehead product of a $CW$-complex whose non-trivial homotopy groups are of dimensions $n$ and $2n-1$ $(n>1)$ (cf. Proposition 7 of [2]).

The arguments through his paper are true for a connected $CW$-complex $Y$ with the following properties:

1) the product $Y \times Y$ is a $CW$-complex whose cells are of the form $E^p \times E^q$ for $p$-cell $E^p$ and $q$-cell $E^q$ of $Y$,

2) for any integer $m$ there exists a $CW$-complex $X \supset Y$ such that $X$ satisfies the property 1) and the inclusion map induces isomorphisms $\pi_i(X) \approx \pi_i(Y)$ for $1 \leqq i < m$ and $\pi_i(X) = 0$ for $i \geqq m$.

In his paper, it is assumed that $Y$ is a connected locally finite $CW$-complex. But this may be replaced by a weaker assumption that $Y$ is a connected countable $CW$-complex[2]. For, if $Y$ is a connected countable $CW$-complex, then, by Theorem (1.9) of [5], $Y$ has the property 1). On the other hand, by Theorem 13 in § 9 of [7], $Y$ is of the same homotopy type as a locally finite simplex $Y'$. Hence $Y'$ is connected and so countable[2]. Therefore, using the simplicial approximation theorem we may easily prove that the elements of $\pi_i(Y) \approx \pi_i(Y')$ for each $i$ are countable. Thus we can construct a countable $CW$-complex $X \supset Y$ such that $\pi_i(X) \approx \pi_i(Y)$ $(1 \leqq i < m)$ and $\pi_i(X) = 0$ $(i \geqq m)$. Since $X$ is countable, its has the property 1). Thus properties 1) and 2) are satisfied for any connected countable $CW$-complex.

In § 2 we shall prove that Proposition 7 of [2] is also true for any $CW$-complex and so for any space whose first two non-trivial homotopy groups are of dimensions $n$ and $2n-1(n>1)$.

In § 3, combining this proposition with results on $H(\Pi, n)$ due to Eilenberg-MacLane [3], we shall give results on the realizability of a given homo-

1) Numbers in brackets refer to the references at the end of the paper.
2) The fact that a connected locally finite $CW$-complex is countable is noticed in p. 223 of [7].