ON THE DETERMINATION OF THE JUMP OF A FUNCTION BY ITS FOURIER COEFFICIENTS

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1. Let $f(x)$ be integrable in the sense of Lebesgue over the interval $(-\pi, \pi)$ and be periodic out side with period 2π. Let the Fourier series of $f(x)$ be

$$\frac{1}{2}a_0 + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx). \tag{1. 1}$$

Then the conjugate series of (1. 1) is

$$\sum_{n=1}^{\infty}(b_n \cos nx - a_n \sin nx) = \sum_{n=1}^{\infty} B_n(x). \tag{1. 2}$$

Let

$$\psi(t) \equiv f(x+t) - f(x-t) - D, \qquad D \equiv D(x). \tag{1. 3}$$

Szasz [2] gave the following theorem for the determination of the jump of a function by its Fourier coefficients:

THEOREM A. *If there exists a number* $D \equiv D(x)$ *such that*

$$\int_0^t \psi(t)dt = o(t) \qquad \textit{and} \qquad \int_0^t |\psi(t)|dt = O(t) \tag{1. 4}$$

as $t \to 0$, *then*

$$\lim_{n\to\infty} \{\overline{S}_{2n}(x) - \overline{S}_n(x)\} = \frac{1}{\pi}\log 2 \cdot D(x),$$

where $\overline{S}_n(x)$ *is the sequence of arithmetic means of the partial sums of the conjugate series.*

Theorem A was further generalized by Chow [1] in the following form:

THEOREM B. *Under the same hypothesis as in Theorem* A

$$\lim_{n\to\infty} \{\overline{S}_{2n}^{\alpha}(x) - \overline{S}_n^{\alpha}(x)\} = \frac{1}{\pi}\log 2 \cdot D(x)$$

for $\alpha > 0$, *where* $\overline{S}_n^{\alpha}$ *is the nth Cesàro mean of order* α *of the conjugate*