

ON INFINITESIMAL HOLOMORPHICALLY PROJECTIVE TRANSFORMATIONS IN KÄHLERIAN MANIFOLDS

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Recently, T. Ōtsuki and Y. Tashiro [9]¹⁾ have studied holomorphically projective correspondences of Kählerian manifolds. Further, Y. Tashiro [11] has introduced the notion of such a correspondence of almost-complex manifolds endowed with a symmetric φ -connection, i. e. a symmetric affine connection with respect to which the almost-complex structure is covariant constant. He has defined the holomorphically projective curvature tensor $P_{kji}{}^h$ which is invariant under holomorphically projective correspondences, and characterized a Kählerian manifold of constant holomorphic sectional curvature by the condition $P_{kji}{}^h = 0$. One of the present authors²⁾ has introduced the notion of the holomorphically projective changes of a φ -connection of some type, called a half-symmetric φ -connection, and the notion of the infinitesimal holomorphically projective transformation, which will be briefly called an *HP*-transformation.

We shall devote this paper to *HP*-transformations in Kählerian manifolds of some types. In § 1, we shall give some preliminary facts concerning Kählerian manifolds and infinitesimal transformations for the later use. We shall characterize in § 2 the analytic *HP*-transformation as an infinitesimal transformation preserving holomorphically planar curves. In § 3, we shall discuss the properties of analytic *HP*-transformations.

T. Sumitomo [10] and K. Yano and T. Nagano [13] have recently studied infinitesimal projective transformations in a Riemannian manifold and obtained valuable results. We shall consider analogous problems concerning *HP*-transformations. In § 4, we shall deal with a Kählerian manifold admitting an analytic *HP*-transformation which leaves the covariant derivative of the holomorphically projective curvature tensor. We shall prove in § 5 that a Kählerian manifold which satisfies $\nabla_k R_{ji} = 0$ and admits a non-trivial analytic *HP*-transformation is necessarily an Einstein one.

As will be proved in § 5, the existence of a non-trivial analytic *HP*-transformation in a Kählerian manifold satisfying $\nabla_k R_{ji} = 0$ reduces the manifold to an Einstein one. So, it might become a problem to investigate

1) The number in brackets [] refers to the Bibliography at the end of the paper.

2) Ishihara, S. [2].