ON REALIZATIONS OF SOME WHITEHEAD PRODUCTS

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Introduction. For any arcwise connected space B with a base point b_0, the sequence of homotopy groups of (B, b_0):

$$\pi_1, \pi_2, \cdots\cdots, \pi_n, \cdots\cdots$$

are defined. These groups except the first one are abelian and are written additively, while the fundamental group π_1 is in general non-abelian and is written multiplicatively. Among these groups there are two kinds of important operations defined topologically. The first one is the operations of π_1 on π_p with $p \geqq 2$ (for the definition see § 16 of [17][1]), i. e. π_p becomes a π_1-modules, namely, for $w \in \pi_1$ and $\alpha \in \pi_p$, $p \geqq 2$, a unique element $w \cdot \alpha$ is determined and

$$w \cdot (\alpha_1 + \alpha_2) = w \cdot \alpha_1 + w \cdot \alpha_2,$$
$$w_1 \cdot (w_2 \cdot \alpha) = (w_1 w_2) \cdot \alpha, \qquad 1 \cdot \alpha = \alpha.$$

The second one is so-called Whitehead products (for the definition see [24]), i. e. for $\alpha \in \pi_p$, $\beta \in \pi_q$ with $p, q \geqq 2$, a bilinear product $[\alpha, \beta] \in \pi_{p+q-1}$ is defined. Hence these products define homomorphisms from $\pi_p \otimes \pi_q$ into π_{p+q-1}, which will be denoted by $W_{p,q}$ or $W_{p,q}(B)$, where the tensor product is taken over the integer coefficients.

It is well-known that these operations satisfy the following properties ([24], [16]):

(1) The skew symmetric law:

$$[\alpha, \beta] = (-1)^{pq}[\beta, \alpha], \text{ or}$$
$$W_{p,q}(\alpha \otimes \beta) = (-1)^{pq} W_{q,p}(\beta \otimes \alpha),$$

(2)
$$w \cdot [\alpha, \beta] = [w \cdot \alpha, w \cdot \beta], \text{ or}$$
$$w \cdot W_{p,q}(\alpha \otimes \beta) = W_{p,q}((w \cdot \alpha) \otimes (w \cdot \beta)),$$

(3) The Jacobi identity:

$$(-1)^{p(r-1)}[\alpha, [\beta, \gamma]] + (-1)^{q(p-1)}[\beta, [\gamma, \alpha]]$$

1) Numbers in brackets refer to the references at the end of the paper.