THE QUOTIENT OF FINITE EXPONENTIAL SUMS*

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A finite sum of the form

$$(1)\qquad \sum_{j=1}^{m} a_j(z)e^{\mu_j z},\ \mu_k \neq \mu_j \text{ for } k \neq j,\ \operatorname{Re}\mu_1 \geqq \operatorname{Re}\mu_2 \geqq \cdots\cdots \geqq \operatorname{Re}\mu_m,$$

will hereafter be called a function of the *class* P if $a_1(z), a_2(z), \ldots\ldots, a_m(z)$ are polynomials in z. If $a_1(z), a_2(z), \ldots\ldots, a_m(z)$ are rational functions then we shall say that (1) belongs to the *class* R. (1) is called an *exponential polynomial* if $a_1(z), a_2(z), \ldots\ldots, a_m(z)$ are all constants.

The following theorem was proved by Lax [2].

THEOREM A. *If $A(z)$ and $B(z)$ are two exponential polynomials*

$$A(z) = \sum_{j=1}^{m} a_j e^{\mu_j z},\quad B(z) = \sum_{j=1}^{n} b_j e^{\nu_j z},$$

and if $f(z) = A(z)/B(z)$ is an entire function then $f(z)$ too is an exponential polynomial.

The example $(e^z - 1)/ze^z$ shows that if $A(z)$ and $B(z)$ both belong to the class P and the quotient $f(z) = A(z)/B(z)$ is an entire function then $f(z)$ need not be a function of the class P. The following theorem is however true.

THEOREM 1. *If*

$$A(z) = \sum_{j=1}^{m} p_j(z)e^{\mu_j z},\quad B(z) = \sum_{j=1}^{n} q_j(z)e^{\nu_j z}$$

(where $p_j(z)$ and $q_j(z)$ are polynomials) are two functions of the class P and the quotient $f(z) = A(z)/B(z)$ is an entire function then $f(z)$ belongs to the class R

Since the quotient of two functions of the class R can be written as the quotient of two functions of the class P we can state the following more general

THEOREM 2. *If $A(z)$ and $B(z)$ are two functions of the class R and*

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