AN EXTENSION OF LINDELÖF'S THEOREM TO MEROMORPHIC FUNCTIONS

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1. Introduction. If $f(z)$ be an entire function of finite order ρ, then Lindelöf has obtained a set of conditions in order that $f(z)$ may be of maximum, mean or minimum type [2; 1, 25-30]. A theorem of the similar nature for meromorphic functions is stated by Valiron [8] and Hari Shanker has recently [3] extended the results of Lindelöf by taking comparison function $r^\rho L(r)$. In this note we prove three theorems which will include the theorems of Lindelöf, Valiron and Hari Shanker as special cases.

Let $f(z)$ be a meromorphic function of finite order ρ. We have

$$(1.1) \qquad f(z) = z^k \exp(Cz^{p_3} + \cdots) \prod_1^\infty E(z/a_n, p_1) / \prod_1^\infty E(z/b_n, p_2),$$

where $Q(z) = Cz^{p_3} + \cdots\cdots$ is a polynomial of degree $p_3 \leqq [\rho]$. Write

$$n(r) = n(r, 1/f) + n(r,f), \; N(r) = N(r, 1/f) + N(r,f).$$

When $\rho > 0$, and $N(r)$ is of order ρ, we define a proximate order $\rho(r)$ for $N(r)$ as follows.

(i) $\rho(r)$ is differentiable for $r > r_0$, except at isolated points at which $\rho'(r-0)$ and $\rho'(r+0)$ exist.

(ii) $$\lim_{r\to\infty} \rho(r) = \rho.$$

(iii) $$\lim_{r\to\infty} r\rho'(r)\log r = 0.$$

(iv) $$\limsup_{r\to\infty} N(r)/r^{\rho(r)} = 1.$$

For the existence of a proximate order see [6] where $\rho(r)$ is constructed with $\log M(r)$; the argument given there can be utilised to construct $\rho(r)$ with the above properties (i)–(iv).

When ρ is integer, we can write $f(z)$ in the form

$$(1.2) \qquad f(z) = z^k \exp(cz^\rho + \cdots) \prod_1^\infty E(z/a_n, \rho) / \prod_1^\infty E(z/b_n, \rho)$$

$$= z^k \exp(cz^\rho + \cdots) P_1/P_2 \text{ (say)}$$

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