A NOTE ON ABSOLUTE CESÀRO SUMMABILITY OF FOURIER SERIES

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(Received December 2, 1959)

1. Let $f(x)$ be an integrable function in Lebesgue sense, and periodic of period 2π, and let

$$f(x) \sim \frac{1}{2}a_0 + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx),$$

$$\sigma_n^\alpha(x) = \frac{1}{2}a_0 + \sum_{\nu=1}^{n} A_{n-\nu}^\alpha(a_\nu \cos \nu x + b_\nu \sin \nu x)/A_n^\alpha,$$

where $\alpha > -1$, and $A_n^\alpha = \binom{\alpha+n}{n}$.

DEFINITION 1. If $\alpha > -1$, and

$$\sum_{n=1}^{\infty} |\sigma_n^\alpha(x) - \sigma_{n-1}^\alpha(x)| < \infty,$$

then the Fourier series of $f(t)$ is said to be absolutely summable (C, α), or briefly summable $|C,\alpha|$ at the point x.

Various theorems concerning the absolute Cesàro summability of Fourier series have been obtained by many authors.

Supposing that $p \geqq 1$ and $f \in L^p$, we write

$$w_p(t) = \left(\frac{1}{2\pi}\int_0^{2\pi} |f(x+t) - f(x)|^p dx\right)^{1/p} \qquad (t > 0). \tag{1. 1}$$

Recently, Chow [3] has proved that

(I) If $1 \leqq p \leqq 2$, $f \in L^p$, and

$$\int_0^{\pi} \frac{w_p(t)}{t} dt < \infty, \tag{1. 2}$$

then the Fourier series of f is summable $|C, \alpha|$ almost everywhere for $\alpha > 1/p$.

(II) If $1 \leqq p \leqq 2$, $f \in L^p$, and

$$w_p(t) = O\left(\log\frac{1}{t}\right)^{-(1+1/p+\epsilon)} \qquad (t \to 0),$$