ANALYTIC TENSOR AND ITS GENERALIZATION

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In our previous papers [7], [8],[1] the notion of almost-analytic vector was introduced in certain almost-Hermitian spaces. In this paper we shall deal with tensors and obtain the notion of Φ-tensors which contains, as special cases, the one of analytic tensors and decomposable tensors.

1. Let us consider an n-dimensional space[2] which admits a tensor field $\varphi_i{}^j$ of type (1, 1). Let $\xi_{(i)}{}^{(j)} \equiv \xi_{i_p \cdots i_1}{}^{j_q \cdots j_1}$ be a tensor of type (q, p). If it commutes with $\varphi_i{}^j$, then we shall say that $\xi_{(i)}{}^{(j)}$ is pure with respect to the corrresponding indices, namely it is pure with respect to i_k and j_h, if

$$(1) \qquad \xi_{i_p \cdots r \cdots i_1}{}^{(j)} \varphi_{i_k}{}^r = \xi_{(i)}{}^{j_q \cdots r \cdots j_1} \varphi_r{}^j{}_h$$

and pure with respect to i_k and i_h, if

$$\xi_{i_p \cdots r \cdots i_k \cdots i_1}{}^{(j)} \varphi_{i_k}{}^r = \xi_{i_p \cdots i_k \cdots r \cdots i_1}{}^{(j)} \varphi_{i_h}{}^r.$$

If $\xi_{(i)}{}^{(j)}$ anti-commutes with $\varphi_i{}^j$ then we shall say that it is hybrid with respect to the corresponding indices. Thus if

$$(2) \qquad \xi_{i_p \cdots r \cdots i_1}{}^{(j)} \varphi_{i_k}{}^r = - \xi_{(i)}{}^{j_q \cdots r \cdots j_1} \varphi_r{}^{j_k},$$

for example, holds good, then it is hybrid with respect to i_k and j_h. $\xi_{(i)}{}^{(j)}$ is called pure (resp. hybrid) if it is pure (resp. hybrid) with respect to all its indices.

$\varphi_i{}^j$ itself and δ_i^j are examples of the pure tensor. If $\varphi_i{}^j$ is a regular tensor i.e. $\det(\varphi_i{}^j) \neq 0$, then the tensor whose components are given by the elements of the inverse matrix of $(\varphi_i{}^j)$ is also pure.

LEMMA 1. *If $\xi_{(i)}{}^{(j)}$ is pure (hybrid) with respect to some indices, then so is $\overset{*}{\xi}_{(i)}{}^{(j)} = \xi_{i_p \cdots i_2 r}{}^{(j)} \varphi_{i_1}{}^r$.*

We shall prove only the case when $\xi_{(i)}{}^{(j)}$ is pure with respect to i_1 and i_k $(k \neq 1)$. In fact, we have

$$\overset{*}{\xi}_{i_p \cdots r \cdots i_1}{}^{(j)} \varphi_{i_k}{}^r = \xi_{i_p \cdots r \cdots i_2 t}{}^{(j)} \varphi_{i_k}{}^r \varphi_{i_1}^t = \xi_{p \cdots i_k \cdots i_2 r}{}^{(j)} \varphi_t{}^r \varphi_{i_1}^t$$

1) The number in brackets refers to the bibliography at the end of the paper.

2) We shall mean by a space a differentiable manifold of class C^∞, and denote by x^i its local coordinates. Indices run over 1, 2, ...n.