# ON ALMOST COMPLEX SYMPLECTIC MANIFOLDS AND AFFINE CONNECTIONS WITH RESTRICTED HOMOGENEOUS HOLONOMY GROUP $Sp(n, C)$

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The purpose of this paper is at first to characterize a $4n$-dimensional affinely connected manifold (with or without torsion) whose restricted homogeneous holonomy group is the real representation of the complex symplectic group $Sp(n, C)$ or one of its subgroups. And conversely, we discuss to introduce in a $4n$-dimensional manifold an affine connection (with or without torsion) whose restricted homogeneous holonomy group is the real representation of $Sp(n, C)$ or one of its subgroups.

The *almost complex symplectic manifold* is equivalent to an almost quaternion manifold (§ 3), but the *natural affine connection* (§ 4) in an almost complex symplectic manifold is different from the *natural affine connection* ($(\varphi, \psi)$-connection by Obata's terminology, [5]) in an almost quaternion manifold[2]. They coincide if and only if the affine connection is a metric connection (with or without torsion) with respect to a *related Riemannian metric* (§ 3, Definition).

**1. Preliminary remarks.** Let $C_{2n}$ be a complex $2n$-dimensional linear space. Complex symplectic group $Sp(n,C)$ in $C_{2n}$ is the subgroup of $GL(2n,C)$ leaving invariant a bilinear form $z^s \wedge w^{s+n} = z^s w^{s+n} - z^{s+n} w^s$ [3] where $(z^\alpha)$ and $(w^\alpha)$ $(\alpha = 1, \ldots\ldots, 2n)$ are vectors in $C_{2n}$. Therefore if $M_{2n}$ is a complex $(2n, 2n)$-matrix giving a transformation of $Sp(n,C)$, then $M_{2n} J_{2n} {}^t M_{2n} = J_{2n}$, where ${}^t M_{2n}$ denotes the transpose of $M_{2n}$ and $J_{2n}$ is a matrix such as $J = \begin{pmatrix} 0 & E_n \\ -E_n & 0 \end{pmatrix}$ [4]. Conversely if $M_{2n}$ satisfies the above relation, then it is a matrix giving a transformation of $Sp(n, C)$.

Next, we consider the real representation of $Sp(n, C)$ in a real $4n$-dimensional real linear space $R^{4n}$.

Put $\mathfrak{M} = \begin{pmatrix} M_{2n} & 0 \\ 0 & \overline{M_{2n}} \end{pmatrix}$, where $\overline{M_{2n}}$ denotes the complex conjugate of $M_{2n}$,

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1) We shall show that this manifold must be necessarily an "almost complex symplectic manifold" (§ 3).
2) Cf. Ehresmann [1]: Libermann [3], [4]; Obata [5].
3) $S$ runs from 1 to $n$. In this paper we adopt the summation convention.
4) In this paper, $E_N$ denotes a unit matrix of degree $N$.