THE CAUCHY PROPERTY OF THE GENERALIZED APPROXIMATELY CONTINUOUS PERRON INTEGRAL

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(Received May 20, 1959)

1. Introduction. We say an integral has the Cauchy property, if it satisfies the following condition (C).

(C) If $f(x)$ is defined in $[a, b]$ and is integrable in each interval $[a+\varepsilon, b-\eta]$, where $a < a+\varepsilon < b-\eta < b$ and

$$\lim_{\epsilon, \eta \to 0} \int_{a+\epsilon}^{b-\eta} f(t)dt \qquad (*)$$

exists, then $f(x)$ is integrable in $[a, b]$ and the integral over $[a, b]$ is equal to the above limit.

Both the special and the general Denjoy integrals have this property. M. E. Grimshaw [1] proved that the approximately continuous Perron integral defined by J. C. Burkill [2] satisfies the condition (C) with the approximate limit instead of the ordinary limit in (*).

By the use of a similar method we will show that the corresponding property is possessed by the generalized approximately continuous Perron integral dfiened by G. Sunouchi and M. Utagawa [3].

The writer expresses his thanks to Dr. G. Sunouchi for his suggestions and criticisms.

2. Generalized approximately continuous Perron integral.

DEFINITION 2. 1. *$U(x)$ [$L(x)$] is termed upper [lower] function of a measurable $f(x)$ in $[a, b]$, provided that*

(i) $U(a) = 0$ $[L(a) = 0]$,

(ii) $\underline{AD}\ U(x) > -\infty$ $[\overline{AD}\ L(x) < +\infty]$ *at each point x,*

(iii) $\underline{AD}\ U(x) \geqq f(x)$ $[\overline{AD}\ L(x) \leqq f(x)]$ *at each point x.*

DEFINITION 2. 2. *If $f(x)$ has upper and lower functions in $[a, b]$ and*

$$\text{l. u. b. } L(b) = \text{g. l. b. } U(b),$$

then $f(x)$ is termed integrable in AP-sense or AP-integrable. The common value of the two bounds is called the definite AP-integral of $f(x)$ and