ON SOME PROPERTIES OF π-STRUCTURES ON DIFFERENTIABLE MANIFOLD

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D. C. Spencer [1][1)] considered under the name "complex almost-product structure" the structure on the n-dimensional differentiable manifold V_n defined by giving two differentiable distributions T_1, T_2 which assign two complemented subspaces of dimension $\geqq 1$ in the complexified tangent space T_x^C at each point $x \in V_n$. G. Legrand [2] called such structure as a π-structure and studied it by generalizing most properties of the almost complex structure which can be regarded as a special case of it [3].

In the following, we assume that on the manifold a structure is defined by giving r $(2 \leqq r \leqq n)$ differentiable distributions $T_1, \ldots\ldots, T_r$ which assign r complemented subspaces of dimension $\geqq 1$ in the complexified tangent space $T_x^C(T_x^C = T_1 + \ldots\ldots + T_r:$ direct sum) at each point $x \in V_n$. We call such structure as an r-π-structure if we want to express the number of the distributions explicitly. Whereas we call it simply as a π-structure if we need not (or can not) express the number r definitely. We generalize some properties of π-structure in the sense of Legrand to the r-π-structure.

In this note we assume that the differentiable manifold V_n as well as the distributions $T_1, \ldots\ldots, T_r$ are of class C^∞ unless we state it explicitly. It is also assumed that the manifold is arc-wise connected and the second countability axiom is satisfied.

1. Fundamental tensor of the π-structure. Suppose the differentiable manifold V_n has a π-structure defined by r differentiable distributions $T_1, \ldots\ldots, T_r$. Let the projection operations from T_x^C to T_α be denoted as $\mathfrak{P}_\alpha$, then we have

$$\mathfrak{P}_\alpha^2 = \mathfrak{P}_\alpha, \quad \mathfrak{P}_\alpha \mathfrak{P}_\beta = 0 \ (\alpha \neq \beta), \tag{1.1}$$

$$\mathfrak{P}_1 + \ldots\ldots + \mathfrak{P}_r = \mathfrak{J}, \tag{1.2}$$

where $\mathfrak{J}$ denotes the identity transformation and the Greek indices vary from 1 to r. Define a transformation $\mathfrak{F}$ on T_x^C by the following:

$$\mathfrak{F} v = \lambda \sum_\alpha w_\alpha \mathfrak{P}_\alpha v, \tag{1.3}$$

1) Numbers in bracket refer to the reference at the end of the paper.