MULTIPLICATIONS IN POSTNIKOV SYSTEMS AND THEIR APPLICATIONS

HARUO SUZUKI

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Introduction. The present work concerns the theory of obstructions for Postnikov complexes of one-connected CW-complex to have multiplications and its application to Thom complexes. Multiplications are introduced stepwisely for dimensions of Postnikov complexes and the obstruction is a set of cohomology classes (See sections 1 and 2). The theory is applicable for $M(O(n))$ and results for a sum of a realizable classes with coefficients in Z_2 are obtained. We compute actually the obstructions for Postnikov complexes of $M(O(2))$ to be an H-space (See section 4). Parallel considerations are made for $M(SO(n))$ and sum of realizable classes with coefficients in Z or Z_p where p is an odd prime number (See section 5).

1. H-spaces. Let A be a topological space. Suppose that a continuous map

$$\mu : A \times A \longrightarrow A \tag{1}$$

is defined and there is the base point $e \in A$ such that

$$\mu(x, e) = x, \ \mu(e, y) = y \tag{2}$$

for any $x, y \in A$. Then A is called an *H-space* and the correspondence $\mu(x, y)$ is called a *multiplication*, which is occasionally denoted by $x{\cdot}y$. A *homotopy commutativity* and *homotopy associativity* are defined in usual ways. One can easily prove

PROPOSITION 1. *A product space of two H-spaces is again an H-space. If given H-spaces are homotopy commutative or homotopy associative, then the product space has the corresponding properties.*

PROOF. Let A_1 and A_2 be H-spaces with multiplication maps μ_1 and μ_2. Define a multiplication μ of $A_1 \times A_2$ by

$$\mu\{(a_1, a_2), (b_1, b_2)\} = \{\mu_1(a_1, b_1), \ \mu_2(a_2, b_2)\} \tag{3}$$

for any $a_1, b_1 \in A_1$ and $a_2, b_2 \in A_2$. Denoting by $e_1 \in A_1$ and $e_2 \in A_2$ respective