RIEMANN-CESÀRO METHODS OF SUMMABILITY V

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(Received March 22, 1960)

1. Introduction. Let s_n^α be the Cesàro sum of a series $\sum_{n=0}^{\infty} a_n$ with $a_0 = 0$, that is, A_n^α being Andersen's notation,

$$s_n^\alpha = \sum_{\nu=0}^{n} A_{n-\nu}^\alpha \, a_\nu,$$

and let σ_n^α be the Cesàro mean of the series $\sum_{n=0}^{\infty} a_n$, that is, $\sigma_n^\alpha = s_n^\alpha / A_n^\alpha$. The series $\sum_{n=0}^{\infty} a_n$ is said to be evaluable (C, α), $\alpha > -1$, to s, if $\sigma_n^\alpha \to s$ as $n \to \infty$. Let $k > 0$ and $\lambda_n = \log(n+1)$. If, when $\omega \to \infty$,

$$\sum_{\lambda_n < \omega} \left(1 - \frac{\lambda_n}{\omega}\right)^k a_n \to s,$$

then the series $\sum_{n=0}^{\infty} a_n$ is said to be evaluable $(\log n, k)$ to s. It is well-known that a series evaluable (C, k) is also evaluable $(\log n, k)$ to the same sum. In the following, let p be a positive integer and let α be a real number such that $\alpha \geqq -1$. The series $\sum_{n=0}^{\infty} a_n$ is said to be evaluable by Riemann-Cesàro method of order p and index α, or briefly, to be evaluable (R, p, α) to s, if the series

$$t^{\alpha+1} \sum_{n=1}^{\infty} s_n^\alpha \left(\frac{\sin nt}{nt}\right)^p$$

converges in some interval $0 < t < t_0$ and its sum tends to $C_{p,\alpha}$ s as $t \to 0+$, where

$$C_{p,\alpha} = \begin{cases} \dfrac{1}{\Gamma(\alpha+1)} \displaystyle\int_0^\infty u^{\alpha-p} (\sin u)^p \, du, & -1<\alpha<p-1 \text{ or } \alpha=0, p=1, \\ 1 \quad , & \alpha = -1. \end{cases}$$

Under this definition, the summabilities $(R, p, -1)$ and $(R, p, 0)$ are the well-known summabilities (R, p) and (R_p), respectively. In our earlier papers [2, 3, 4],