NOTE ON SUSPENSION AND HOPF INVARIANT

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1. Introduction. Let A be a k-connected special complex ($k \geqq 0$), let ${}^nA$ be the n-fold suspension of A, and let ${}^nA * {}^nA$ be a join of the two copies of ${}^nA$. James [5] defined the homomorphism

$$h: \pi_{r+1}({}^{n+1}A : C_+, C_-) \to \pi_{r+1}({}^nA * {}^nA),$$

using a canonical isomorphism and a combinatorial extension of the reduced product space. And, he defined Hopf invariant $H = h \circ i$ on the special complex, where i is the inclusion homomorphism in the triad sequence.

On the sphere, the generalized suspension homomorphism agrees with the classical one, but we do not know any relation between the generalized Hopf invariant H and the classical one H_0, for instance, Toda's generalized Hopf invariant (cf. James [6]).

Concerning it, we obtain the following

THEOREM I. *On the sphere S^n, H agrees with H_0 except their signs, if $r \leqq 3n-4$.*

James proved that

$$h: \pi_{r+1}({}^{n+1}A : C_+, C_-) \to \pi_{r+1}({}^nA * {}^nA)$$

is the isomorphism onto, if $r \leqq 3(k+n)+1$.

Then, we have the exact sequence

$$\pi_{3(k+n)+1}({}^nA) \xrightarrow{E} \pi_{3(k+n)+2}({}^{n+1}A) \xrightarrow{H} \cdots\cdots$$
$$\cdots\cdots \xrightarrow{H} \pi_{r+2}({}^nA * {}^nA) \xrightarrow{\Delta'} \pi_r({}^nA) \xrightarrow{E} \pi_{r+1}({}^{n+1}A) \xrightarrow{H}$$
$$\pi_{r+1}({}^nA * {}^nA) \to \cdots\cdots.$$

And ${}^nA * {}^nA$ is $(2(k+n)+2)$-connected.

Therefore, as is well known, the generalized suspension homomorphism

$$E: \pi_r({}^nA) \to \pi_{r+1}({}^{n+1}A)$$

is onto isomorphism, if $r \leqq 2(k+n)$, and onto homomorphism, if $r = 2(k+n)+1$, and if $r \leqq 3(k+n)+1$, then

$$\text{image } E = \text{kernel } H.$$